2 UNITS AND CONSTANTS

2.1 Mathematical Constants

Table 19 gives some important mathematical constants. These and other constants are given to greater accuracy in Table 1 of Sec. 1-2.

Table 19. Mathematical Constants

π	π^2	$1/\pi$	$2/\sqrt{\pi}$*	e
3.14159	9.8696	0.31831	1.1284	2.71828
$1/e$	$M = \log e$	$1/M = \ln 10$	$\ln 2$	First zero of $J_0(x)$
0.36788	0.43429	2.3026	0.69314	2.4048

* Factor used in correcting for Maxwell distribution.

2.2 Engineering Units and Conversion Factors

Table 20 gives conversion factors for engineering units. The column "More exact value" includes definitions (marked with asterisks) and equivalents calculated from definitions.

Table 20. Conversion Factors for Engineering Units

Unit	Usual engineering approximation	More exact value	Reference to Sec. 1-3: Definition	Reference to Sec. 1-3: Other units and conversion factors
Length				Table 4
1 cm	0.3937 in.	0.3937*†	Art. 2.11	
1 in	2.54 cm	2.540005	Art. 2.11	
Area				Table 5
1 cm^2	0.001076 ft^2	0.001076387		
1 $in.^2$	6.45 cm^2	6.451626		
1 ft^2	929 cm^2	929.0275		
Volume				Table 6
1 cm^3	3.53×10^{-5} ft^3	3.531445×10^{-5}		
1 ft^3	2.83×10^4 cm^3	28,317.02		
1 l	1,000 cm^3	1,000.028	Table 6	
1 U.S. gal	231 $in.^3$	231*	Table 6	
Mass				Table 7
1 kg	2.205 lb	2.204622	Art. 2.12	
1 lb	453.6 g	453.5924277*†	Art. 2.12	
Time				Table 8
1 day	86,400 sec.	86,400* (mean solar day)	Art. 2.13	
1 year	3.16×10^7 sec	3.155695×10^7	Art. 2.13	
Force				Table 9
Standard gravity, g_0	32.2 ft/sec^2	32.1740	Art. 2.14	
Standard gravity, g_0	981 cm/sec^2	980.665*	Art. 2.14	
Density				Table 13
1 g/cm^3	62.4 lb/ft^3	62.42833		
1 lb/ft^3	0.0160 g/cm^3	0.01601837		
Pressure				Table 14
1 atmosphere	14.7 psi	1,013,250* dynes/cm^2	Table 14	
Temperature				Table 18
t_F	$\frac{9}{5}t_C + 32$			
t_C	$\frac{5}{9}(t_F - 32)$			
Absolute zero	−273°C	−273.16*	Art. 2.21	
Absolute zero	−460°F	−459.69	Art. 2.21	
Work, energy, heat				Table 15
1 joule = 1 watt-sec	10^7 ergs	10^7*	Art. 2.3	
1 I.T. cal	4.187 joules	4.18684	Art. 2.23	
1 I.T. cal	0.00397 Btu	0.00396832	Art. 2.23	
1 Btu	252 cal	251.996	Art. 2.23	
1 Mev	1.60×10^{-13} joule	1.60206×10^{-13}	Art. 4.32	Table 28
Power				Table 16
1 kw	3412 Btu/hr	3412.10	Art. 2.3	
Heat flux and power density				
1 cal/(sec)(cm^2)	13,270 Btu/(hr)(ft^2)	13,272.1		Table 22
1 watt/cm^2	3,170 Btu/(hr) (ft^2)	3,170.20		Table 22
1 watt/cm^3 ≈ 1kw/l	96,600 Btu/(hr)(ft^3)	96,620.4		Table 23

* Exact value by definition.

† United States definition. Other measures involving this unit are based on this definition.

2.3 Physical Constants and Conversion Factors

For more exact values and additional data, see Art. 4 of Sec. 1-3.

2.31 Physical Constants. Table 21 gives the physical constants most used in nuclear engineering (see Art. 4.2 of Sec. 1-3).

2.32 Mass and Energy and Wavelength

$$1 \text{ amu} = 1.660 \times 10^{-24} \text{ g} = 931 \text{ Mev}$$
$$\text{Mass of electron} = 0.00055 \text{ g} = 0.51 \text{ Mev}$$

For masses of particles, energy equivalents, and wavelengths, see Art. 4.32 of Sec. 1-3.

Table 21. Physical Constants

Constant	Usual engineering approximation	Best value	Reference to Sec. 1-3
Avogadro's number N, chemical scale	0.602×10^{24} (g-mole)$^{-1}$	0.602322×10^{24}	Table 27
De Broglie wavelength of neutron	$2.86 \times 10^{-9}/E^{1/2}$ cm (E in ev)	$2.86005 \times 10^{-9}/E^{1/2}$	Table 30
Boltzmann's constant k	8.62×10^{-5} ev/°C 4.79×10^{-5} ev/°F	8.6167×10^{-5} 4.7871×10^{-5}	Table 27 Table 27
Electronic charge e	1.60×10^{-19} coulomb 4.80×10^{-10} esu	1.60206×10^{-19} 4.80286×10^{-10}	Table 27 Table 27
Energy of 2,200-m/sec neutron	0.0253 ev	0.0252973	Table 30
kT temperature for 2,200-m/sec neutrons	20°C (69°F)	20.426°C(68.767°F)	Table 30
Molar volume of ideal gas at 1 atm., chemical scale	22.4×10^3 cm^3-atm./g-mole 359 ft^3-atm./lb-mol	22.4146×10^3 358.746	Table 27 Table 27
Physical scale/chemical scale	1.000	1.000272	Table 27 (footnote) and Art. 4.31
Velocity of light c	3.00×10^{10} cm/sec	2.997930×10^{10}	Table 27

3 THE ELEMENTS AND NUCLIDES

3.1 Atomic Weights and Atomic Numbers

Table 22 gives atomic weights and atomic numbers of the elements. Tables 1 and 2 of Sec. 11 give, respectively, the periodic chart and important physical properties of the elements.

Table 22. Atomic Weights and Atomic Numbers of the Elements

Element	Symbol	Atomic number	Atomic weight*	Element	Symbol	Atomic number	Atomic weight*
Actinium	Ac	89	227	Mercury	Hg	80	200.61
Aluminum	Al	13	26.89	Molybdenum	Mo	42	95.95
Americium	Am	95	(243)	Neodymium	Nd	60	144.27
Antimony	Sb	51	121.76	Neon	Ne	10	20.183
Argon	A	18	39.944	Neptunium	Np	93	(237)
Arsenic	As	33	74.91	Nickel	Ni	28	58.71
Astatine	At	85	(210)	Niobium	Nb	41	92.91
Barium	Ba	56	137.36	Nitrogen	N	7	14.008
Berkelium	Bk	97	(249)	Nobelium	No	102	(253?)
Beryllium	Be	4	9.013	Osmium	Os	76	190.2
Bismuth	Bi	83	209.00	Oxygen	O	8	16.000
Boron	B	5	10.82	Palladium	Pd	46	106.4
Bromine	Br	35	79.916	Phosphorus	P	15	30.975
Cadmium	Cd	48	112.41	Platinum	Pt	78	195.09
Calcium	Ca	20	40.08	Plutonium	Pu	94	(244)
Californium	Cf	98	(249)	Polonium	Po	84	210
Carbon	C	6	12.011	Potassium	K	19	39.100
Cerium	Ce	58	140.13	Praseodymium	Pr	59	140.92
Cesium	Cs	55	132.91	Promethium	Pm	61	(145)
Chlorine	Cl	17	35.457	Protactinium	Pa	91	231
Chromium	Cr	24	52.01	Radium	Ra	88	226.05
Cobalt	Co	27	58.94	Radon	Rn	86	222
Copper	Cu	29	63.54	Rhenium	Re	75	186.22
Curium	Cm	96	(245)	Rhodium	Rh	45	102.91
Dysprosium	Dy	66	162.51	Rubidium	Rb	37	85.48
Einsteinium	Es	99	(254)	Ruthenium	Ru	44	101.1
Erbium	Er	68	167.27	Samarium	Sm	62	150.35
Europium	Eu	63	152.0	Scandium	Sc	21	44.96
Fermium	Fm	100	(252)	Selenium	Se	34	78.96
Fluorine	F	9	19.00	Silicon	Si	14	28.09
Francium	Fr	87	(223)	Silver	Ag	47	107.880
Gadolinium	Gd	64	157.26	Sodium	Na	11	22.991
Gallium	Ga	31	69.72	Strontium	Sr	38	87.63
Germanium	Ge	32	72.60	Sulfur	S	16	32.066
Gold	Au	79	197.0	Tantalum	Ta	73	180.95
Hafnium	Hf	72	178.50	Technetium	Tc	43	(99)
Helium	He	2	4.003	Tellurium	Te	52	127.61
Holmium	Ho	67	164.94	Terbium	Tb	65	158.93
Hydrogen	H	1	1.0080	Thallium	Tl	81	204.39
Indium	In	49	114.82	Thorium	Th	90	232.05
Iodine	I	53	126.91	Thulium	Tm	69	168.94
Iridium	Ir	77	192.2	Tin	Sn	50	118.70
Iron	Fe	26	55.85	Titanium	Ti	22	47.90
Krypton	Kr	36	83.80	Tungsten	W	74	183.86
Lanthanum	La	57	138.92	Uranium	U	92	238.07
Lead	Pb	82	207.21	Vanadium	V	23	50.95
Lithium	Li	3	6.940	Xenon	Xe	54	131.30
Lutetium	Lu	71	174.99	Ytterbium	Yb	70	173.04
Magnesium	Mg	12	24.32	Yttrium	Y	39	88.92
Manganese	Mn	25	54.94	Zinc	Zn	30	65.38
Mendelevium	Md	101	(256)	Zirconium	Zr	40	91.22

* A value given in parentheses denotes the mass number of the isotope of longest known half-life.

3.2 The Nuclide Chart

Tables giving important properties of the nuclides are listed in Tables 4 and 5 of this section. Table 23 is a chart giving selected properties of the naturally occurring nuclides and their neutron-activation products.

3.21 Use of Table 23. This chart gives the atomic number, atomic weight, and neutron absorption cross sections of the elements; the isotopic abundance and neutron absorption cross section of the naturally occurring isotopes; and the half-life and modes of decay of naturally radioactive and neutron-activated product nuclides.

Table 23. A Chart of Nuclide Properties

Key to chart Atomic number and symbol. Consult alphabetic list of elements (Table 22).

Element in natural isotopic abundance: $_1H$ — 1.0080 (Atomic weight, chemical scale); σ .33 (Absorption cross section, barns); Atomic number and symbol

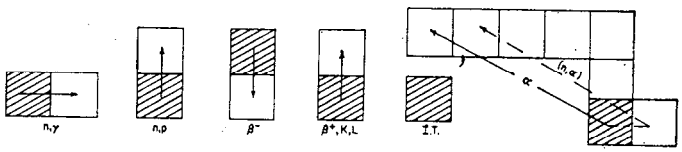
Naturally occurring nuclide (heavy box): 40 K, * 0.0119, σ 70 — Mass number; Data on radioactivity in supplementary box; Isotopic abundance, per cent; Absorption cross section, barns

Supplementary data for naturally occurring nuclide: K^{40} 1.3 X 10^9 y, β^-, K — Half-life; Mode of decay

Artificial nuclide (light box): 3; * 12.4y; β^- — Mass number; Additional data in supplementary box; Half-life (y=years, d=days, h=hours, m=minutes, s=seconds); Mode of decay

Cross section σ = absorption cross section (n,γ except where otherwise stated). σ_f = fission cross section (σ includes σ_f). All cross sections are for 2200 m/sec except as noted in Art. 3.21 of text. PN = pile neutrons.

Modes of decay



$_1H$	1.0080; σ .33
$_2He$	4.003; σ .007
$_3Li$	6.940; σ 71
$_4Be$	9.013; σ .010
$_5B$	10.82; σ 755
$_6C$	12.011; σ .0032
$_7N$	14.008; σ 1.9
$_8O$	15.000; σ <.0002
$_9F$	19.00; σ .009
$_{10}Ne$	20.183; σ < 1
$_{11}Na$	22.991; σ .53
$_{12}Mg$	24.32; σ .063
$_{13}Al$	26.98; σ .23
$_{14}Si$	28.09; σ .13
$_{15}P$	30.975; σ .20
$_{16}S$	32.066; σ .49

Element	1	2	3	4
H	99.985; σ .33	0.015; σ .00057	12.26y; β^-	
He	.00013; $\sigma_{n,p}$ 5400		~100; 0	

Element	6	7	8	9	10	11	12	13	14
Li	7.5; $\sigma_{n,\alpha}$ 950	92.5; σ .033	0.84s; β^-						
Be			<4X10⁻⁶; $\sigma_{n,\alpha}$ 2α	100; σ .010	2.7X10⁶y; β^-				
B						18.8; $\sigma_{n,\alpha}$ 4020	81.2; σ < .05	0.025s; β^-	
C								98.89; σ .0032	1.11; σ .0009	5600y; β^-

Element	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
N	*99.63; σ 1.85	0.37; .00002	7.4s; β^-	4.14s; β^-											
O			99.759; <.00002	0.037; $\sigma_{n,\alpha}$ .5	0.204; .00022	29s; β^-									
F							100; σ .009	11s; β^-							
Ne								90.8	0.26	8.9	40s; β^-				
Na											100; σ .53	15.0h; β^-			
Mg												78.8; σ .03	10.1; σ .27	11.1; σ .03	9.5m; β^-
Al															100; σ .23; 2.30m; β^-

N^{14}: $\sigma_{n,s}$ 1.75; $\sigma_{n,\gamma}$ 0.10

Element	28	29	30	31	32	33	34	35	36	37
Si	92.17; σ .1	4.71; σ .3	3.12; σ .11	2.62h; β^-						
P				100; σ .20	14.5d; β^-	25.1d; β^-				
S					95.0; $\sigma_{n,\alpha}$ .002	0.75; $\sigma_{n,p}$ .002	4.2; σ .26	87d; β^-	0.017; σ .14	5.0m; β^-

Table 23. A Chart of Nuclide Properties. *(Continued)*

Element	Z	Mass	Isotope	Abundance	Half-life	Decay	σ
S	16	32.066					σ .49
			32	95.0			$\sigma_{n,\alpha}$ .002
			33	0.75			$\sigma_{n,p}$ .002
			34	4.2			σ .26
			35		87d	β^-	σ .14
			36	0.017			
			37		5.0m	β^-	
Cl	17	35.457					σ .33
			35	75.53			σ 44; $\sigma_{n,p}$ .30
			36		3.1×10^5y	β^-,(K)	
			38		24.47	β^-	σ .56
			38		37.3m	β^-	
A	18	39.944					σ .6
			35	0.337			σ .6
			37		35d	K,L	
			38	0.063			σ .8
			39		260y	β^-	
			40	99.60			σ .53
			41		1.82h	β^-	
K	19	39.100					σ 2.0
			39	93.2			σ 1.9
			40	R0.0119			σ 70
			42		6.8		σ 1.1
					12.5h	β^-	
			K^{40}		1.3×10^9y	β^-,(K)	
Ca	20	40.08					σ .43
			40	96.9			σ .2
			41		1.1×10^5y	K	
			42	0.64			σ .40
			43	0.14			
			44	2.1			σ .6
			45		160d	β^-	
			46	0.0032			σ .3
			47		4.7d	β^-	
			48	0.18			σ 1.1
			49		8.7m	β^-	
Sc	21	44.96					σ 23
			45	100			σ 23
			46		85d	β^-	
Ti	22	47.90					σ 6.0
			47	8.0			σ .6
			48	7.4			σ 1.6
			49	73.8			σ 7.8
			50	5.5			σ 1.8
			51	5.3			σ .14
			51		5.80m	β^-	
V	23	50.95					σ 4.9
			50	0.25			σ ~100
			51	99.75			σ 4.5
			52		3.77m	β^-	
Cr	24	52.01					σ 3.1
			51	4.4			σ .16
			52		27d	K	
			53	83.7			σ .8
			54	9.5			σ .18
			55		3.6m	β^-	σ .37
Mn	25	54.94					σ 13.3
			55	100			σ 13.3
			56		2.58h	β^-	
Fe	26	55.85					σ 2.5
			54	5.9			σ 2.2
			55		2.9y	K	
			57	2.20			σ 2.4
			58	0.33			σ .9
			59		45d	β^-	
Co	27	58.94					σ 37
			59	100			σ 37
			60		5.2y	β^-	
Ni	28	58.71					σ 4.6
			58	68.0			σ 4.3
			59		8×10^4y	K	
			61	1.1			σ 2
			62	3.7			σ 15
			63		80y	β^-	
			64	1.0			σ 2
			65		2.56h	β^-	
Cu	29	63.54					σ 3.7
			63	69.0			σ 4.4
			64		12.8h	K,β^-,β^+	
			65	31.0			σ 2.2
			66		5.1m	β^-	
Zn	30	65.38					σ 1.10
			67	4.1			
			68	R18.6			σ 1.1
			69	R			
						β^-	
			70	0.63			σ .09
			71		5h,2.2m	β^-	
			72				
			48.9				σ .5
			245d			K (β^+)	
			27.8				
			Zn^{68}				σ 0.1; σ 1.0
			Zn^{69}		13.8h I.T.	52m β^-	
Ga	31	69.72					σ 2.9
			69	60.1			σ 1.9
			70		21m	β^-	
			71	39.9			σ 4.6
			72		14.1h	β^-	
Ge	32	72.60					σ 2.3
			73	20.5			σ 3.4
			74		12d	K	
			75	27.4			σ 1.0
			76	7.8			σ 14
			77	36.5			σ .80
			77		12h	β^-	
As	33	74.91					σ 4.3
			75	100			σ 4.3
			76		26.7	β^-	
Se	34	78.96					σ 13
			74	0.93			σ 40
			75		127d	K	
			76	9.1			σ .82
			77	7.5			σ 41
			78	23.6			σ .4
			79		7×10^4y	β^-	
			80	49.9			σ .53
			81	R		β^-	
			82	9.0			σ .054
			83		65s,23m	β^-	
			Se^{81}				57m I.T., 18m β^-
Br	35	79.916					σ 6.6
			79	R50.6			σ 11.4
			80	R		β^-,(K,β^+)	
			81	49.4			σ 2.8
			82	35.9h		β^-	
			Br^{79}				σ 2.9; σ 8.3
			Br^{80}		4.4h I.T.	18.5m β^-(K,β^+)	
Kr	36	83.80					σ 28
			78	0.35			σ 2
			79		34h	K,L,(β^+)	
			80	2.27			σ 90
			81		2×10^5y	K	
			82	11.6			σ 40
			83	11.5			σ 200
			84	57.0			σ .16
			85	10.4y		β^-	
			86	17.3			σ .06
			87		78m	β^-	
			Rb^{87}		4.3×10^{10}y	β^-	
Rb	37	85.48					σ .7
			84	72.2			σ .7
			85		18.6d	β^-	
			86	R27.8			σ .14
			88		18m	β^-	
Sr	38	87.63					σ 1.3
			84	0.55			σ 1
			85		65d	K	
			86	9.8			σ 1.3+
			87	7.0			
			88	82.7			σ .006
			89		54d	β^-	
Y	39	88.92					σ 1.3
			89	100			σ 1.3
			90		64.0h	β^-	
Zr	40	91.22					σ .18
			91	51.5			σ .1
			92	11.2			σ 1
			93	17.1			σ .2
			94		9×10^5y	β^-	
			95	17.4			σ .1
			96	65d		β^-	
			97	2.8			σ .1
					17h	β^-	
Nb	41	92.91					σ 1
			93	100			σ 1+
			94		20,000y	β^-	
Mo	42	95.95					σ 2.5
			92	15.7			σ < 0.3
			93		>2y	K	
			94	9.3			
			95	15.7			σ 14
			96	16.5			σ 1
			97	9.5			σ 2
			98	23.8			σ .45
			99		67h	β^-	
			100	9.5			σ .2
			101		15m	β^-	
Tc	43						

Table 23. A Chart of Nuclide Properties. (*Continued*)

Element	Atomic weight	σ
$_{44}$Ru	101.1	σ 2.5
$_{45}$Rh	102.91	σ 150
$_{46}$Pd	106.4	σ 8
$_{47}$Ag	107.880	σ 60
$_{48}$Cd	112.41	σ3300
$_{49}$In	114.82	σ 190
$_{50}$Sn	118.70	σ .6
$_{51}$Sb	121.76	σ 5.5
$_{52}$Te	127.61	σ 4.6
$_{53}$I	126.91	σ 6.3
$_{54}$Xe	131.30	σ 35
$_{55}$Cs	132.91	σ 31
$_{56}$Ba	137.36	σ 1.2
$_{57}$La	138.92	σ 8.9
$_{58}$Ce	140.13	σ .7
$_{59}$Pr	140.92	σ 11
$_{60}$Nd	144.27	σ 46
$_{61}$Pm		
$_{62}$Sm	150.43	σ 8250
$_{63}$Eu	152.0	σ 4300
$_{64}$Gd	156.9	σ 39,000
$_{65}$Tb	158.93	σ 45
$_{66}$Dy	162.51	σ 1100
$_{67}$Ho	164.94	σ 64
$_{68}$Er	167.27	σ 170

Ru

96	97	98	99	100	101	102	103	104	105
5.6 σ .01	2.9d K	1.9	12.7	12.7	17.0	31.5 σ 1.2	40d β^-	18.6 σ .7	4.5h β^-

Rh^{104} 4.4m I.T., 42s β^-

Rh

103	104
100 σ 150	* β^-

Pd

102	103	104	105	106	107	108	109	110	111
1.0 σ 4.8	17d K	11.0	22.2	27.3	7×10^{6}y β^-	26.7 σ 11	13.6h β^-	11.8 σ .4	* β^-

Pd^{111} 5.5h I.T., 22m β^-

Ag

107	108	109	110
51.4 σ 30	2.3m β^- (K,β^+)	*48.6 σ 113	* β^-

Ag^{109} σ 2.8, σ 110 — Ag^{110} 270d β^-(I.T.), 24s β^-

Cd

106	107	108	109	110	111	112	113	114	115	116	117
1.22 σ 1	6.7h K,(β^+)	0.88	1.3y K,L	12.4 σ .2+	12.8	24.0 σ .03+	12.3 σ 27,000	28.8 σ .14 +1.1	43d β^- 53h β^-	7.6 σ 1.4	3h I.T. 50m β^-

In

113	114	115	116
*4.2 σ 56	* β^-,β^+,K	*95.8 σ 145 +52	54m β^- 13s β^-

In^{113} σ 56, σ 2 — In^{114} 49d I.T.(K), 72s β^-(K,β^+) — In^{115} 6×10^{14}y β^-

Sn

112	113	114	115	116	117	118	119	120	121	122	123	124	125
1.02 1.3	112d K,L	0.69	0.38	14.3 σ .006	7.6	24.1 σ .01+	*8.5	32.5 σ .001 +.1	>1y β^- 27h β^-	4.8 σ .001 +.2	130d β^- 40m β^-	6.1 σ .2 +.004	9.5m β^- 10d β^-

Sn^{119} 275d I.T.

Sb

121	122	123	124
57 σ 7.0	2.8d β^-,(K,β^+)	43 σ 3.5	60d β^-

Te

120	121	122	123	124	125	126	127	128	129	130	131
0.091 σ 70	150d I.T. 17d K	2.5 σ 2.7	0.88 σ 400	4.6 σ 7	7.0 σ 1.5	18.7 σ .09 +.8	110d I.T. 9.3h β^-	31.8 σ .015 +.13	33d I.T. 72m β^-	34.4 σ .2	30h,25m β^-

Te has several long lived I.T.

I

127	128
100 σ 6.3	25.0m β^-,(K)

Xe

124	125	126	127	128	129	130	131	132	133	134	135	136	137
0.094	18h K	0.092	36.4d K	1.92 σ < 5	26.4 σ 45	4.1 σ < 5	21.2 σ 120	26.9 σ < 5	5.27d β^-	10.4 σ < 5	*9.2h β^-	8.9 σ .15	3.8m β^-

Xe^{135} σ 2.72 × 10^6

Cs

133	134
100 σ 29	2.3y β^-

Ba

130	131	132	133	134	135	136	137	138	139
0.101 σ 6	11.6d K	0.097 σ 3	8y K	2.42 σ < 4	6.6 σ 5	7.8 σ < 1	11.3 σ 4	71.7 σ .55	85m β^-

La

138	139	140
*0.089	99.911 σ 8.9	40.2h β^-

La^{138} 2×10^{11}y · β^-

Ce

136	137	138	139	140	141	142	143
0.19 σ .6 +6.3	35h I.T. 9h K	0.26 σ 1	140d K	88.47 σ .6	32d β^-	11.08 σ 1	33h β^-

Pr

141	142
100 σ 11	19.1h β^-

Nd

142	143	144	145	146	147	148	149	150	151
27.1 σ 17	12.2 σ 320	*23.9 σ 5	8.3 σ 44	17.2 σ 2	11.6d β^-	5.7 σ 4	1.8h β^-	5.6 σ 3	15m β^-

Nd^{144} ~2×10^{15}y α

Sm

144	145	146	147	148	149	150	151	152	153	154	155
3.1 σ ~ .03	1.0y K	~5×10^{7}y α	*15.07	11.2	13.8 σ_{PN} 66,000	7.4	80y β^-	26.8 σ 140	47h β^-	22.7 σ 5	23m β^-

Sm^{147} 1.3 × 10^{11}y α

Eu

151	152	153	154
*47.8 σ_{PN} 9000	* β^-,K	52.2 σ_{PN} 420	16y β^-

Eu^{151} σ_{PN} 1400, σ_{PN} 7200 — Eu^{152} 9.3h β^-(K), 13y K(β^-)

Gd

152	153	154	155	156	157	158	159	160	161
0.20 σ < 125	236d K	2.15	14.7 σ_{PN} ~70,000	20.5	15.7 σ_{PN} ~160,000	24.9 σ 4	18h β^-	21.9 σ .8	3.7m β^-

Tb

159	160
100 σ 45	72d β^-

Dy

156	157	158	159	160	161	162	163	164	165
0.052	8.2h K	0.090	134d K,L	2.29	18.9	25.5	25.0	28.2 σ_{PN} 2700	2.32h β^-

Ho

165	166
100 σ 64	27.2h (> 30y) β^-

Er

162	163	164	165	166	167	168	169	170	171
0.136	75m K	1.56	10h K	33.4	22.9	27.1 σ 2	9.4d β^-	14.9 σ 9	7.5h β^-

Table 23. A Chart of Nuclide Properties. *(Continued)*

Element	Mass	Isotope	Abundance	Half-life	Decay
$_{68}$Er	167.27	162	0.136	75m	K
		163		1.56	
		164		10h	K
		165		33.4	
		166		22.9	
		167		27.1	σ 2
		168		9.4d	β^-
		169		14.9	σ 9
		170		7.5h	β^-
		171			
$_{69}$Tm	168.94	168		100	σ 125
		169		129d	β^-
$_{70}$Yb	173.04	168	0.14	32d	K
		169	σ_{PN} 11,000	3.03	
		171		14.3	
		172		21.8	
		173		16.2	
		174		31.8	$\sigma \sim 60$
		175		4.2d	β^-
		176		12.7	σ 7
		177		2.0h	β^-
$_{71}$Lu	174.99	175		97.40	σ 35
		176		^{m}2.60	σ 4000
		177		6.8d	β^-
				Lu^{176} 3×10^{10}y β^-	$Lu^{175}(n,\gamma)Lu^{176}$ 3.7h β^-
$_{72}$Hf	178.50	174	0.18	70d	K
		175	$\sigma \sim 1000$	5.2	$\sigma < 30$
		176		18.5	σ 370
		177		27.1	σ 60
		178		13.8	σ 65
		179		35.2	σ 10
		180		46d	β^-
		181			
$_{73}$Ta	180.95	180	0.012	99.988	σ 22
		181		112d	β^-
$_{74}$W	183.86	180	^{m}0.14	140d	K,L
		181	$\sigma \sim 20$	26.2	σ 20
		183		14.3	σ 11
		184		30.7	σ 2.0
		185		74d	β^-
		186		28.7	σ 36
		187		24h	β^-
		188			
$_{75}$Re	186.22	185		37.1	σ 105
		186		91h	β^-,(K)
		187		^{m}62.9	σ 75
		188		17h	β^-
				W^{180} 10^{14}y ? α	Re^{187} $\sim 5 \times 10^{10}$y β^-
$_{76}$Os	190.2	184	0.018	95d	K,L
		185	$\sigma \sim 20$	1.59	
		186		1.64	
		187		13.3	
		188		16.1	
		189		26.4	σ 8
		190		16d	β^-
		191		41.0	σ 1.6
		192		31h	β^-
		193			
$_{77}$Ir	192.2	190		38.5	σ 960
		191		74d	β^-,(K)
		192		61.5	σ 120
		194		19h	β^-
$_{78}$Pt	195.09	190	^{m}0.012	3.0d	K
		191	$\sigma \sim 90$	0.78	σ 8
		193		<500y	K
		195		32.8	σ 1.2
		196		33.7	σ 27
		197		25.4	σ 1.2
		198		19h	β^-
		199		7.2	σ 4
		200		30m	β^-
$_{79}$Au	197.0	198		100	σ 98
		199		2.70d	β^-
				Pt^{190} $\sim 10^{12}$y α	
$_{80}$Hg	200.61	196	0.15	65h	K
		197	σ_{PN} 3100	10.0	
		198		16.9	σ_{PN} 2500
		199		23.1	$\sigma < 50$
		200		13.2	$\sigma < 50$
		201		29.8	σ 3
		202		46d	β^-
		203		6.8	σ .4
		204		5.2m	β^-
		205			
$_{81}$Tl	204.39	203		29.5	σ 11
		204		4.1y	β^-,(K)
		205		70.5	σ .11
		206		4.20m	β^-
$_{82}$Pb	207.21	204		1.3	σ .9
		205		$>10^8$y	L
		206		26	σ .03
		207		21	σ .73
		208		52	.00045
		209		3.3h	β^-
		210			
$_{83}$Bi	209.00	209		^{m}100	σ .032
		210		^{m}5.6 $\times 10^6$y	α, β
				Bi^{209} 3×10^{17}y (?) α σ .019 σ .013	Bi^{210} 5d $\beta^-(\alpha)$ 2.6×10^6y $\alpha(\beta^-)$
$_{84}$Po		210		138.40d	α
$_{85}$At					
$_{86}$Rn		222		3.825d	α
$_{87}$Fr					
$_{88}$Ra	226.05	226		^{m} σ_{PN} 20	
		227			41m β^-
$_{89}$Ac		227			22y $\beta^-(\alpha)$
				Ra^{226} 1620y α	
$_{90}$Th	232.05	232		^{m}100	σ 7.5
		233		^{m}23.3m	β^-
$_{91}$Pa		233		^{m}27.4d	β^-
$_{92}$U	238.07	233		^{m}1.62 $\times 10^5$y	α
		234		^{m}0.0057	σ 92 (σ_f <685)
		235		^{m}0.714	σ 694 (σ_f 582)
		236		2.39 $\times 10^7$y	α,(f)
		237		6.75d	β^-
		238		^{m}99.28	σ 2.73
		239		23.5m	β^-
				U^{233} σ 588 (σ_f 532)	U^{234} 2.52 $\times 10^5$y α,(f)
				U^{236} σ 6	U^{238} 4.51 $\times 10^9$y α,(f)
				U^{235} 7.1 $\times 10^8$y α,(f)	U^{239} σ_{PN} 37 (σ_f σ_{PN} 15)
				Th^{232} 1.39 $\times 10^{10}$y α,(f)	Th^{233} σ_{PN} 1400 σ_{PN} 15
				Pa^{233} σ 66	
$_{93}$Np		239		^{m}2.33d	β^-
				Np^{239} σ_{PN} 60 (σ_f < 31)	
$_{94}$Pu		239		^{m}24,300y	α,(f)
		240		6600y	α,(f)
		241		13.2y	$\beta^-(\alpha)$
				Pu^{239} 2200 m/sec neutrons σ 1025 (σ_f 738)	Pu^{240} σ 250 (σ_f < 4)
				Pu^{241} σ_{PN} 1480 (σ_f 950)	

The data presented and the method of using the chart are explained in the key at the top of the chart. The following limitations of Table 23 should be noted.

Cross Sections. Cross sections are for 2,200-m/sec neutrons with the following exceptions.

1. Where cross sections are given for the spectrum of pile neutrons, this is denoted by the subscript PN.

2. Where the absorption (or fission) cross section is not $1/v$-dependent, the value has been multiplied by the appropriate factor,[1,*] so that a correct average cross section for the Maxwell distribution will be obtained when the Maxwell-distribution factor $\sqrt{\pi}/2 = 1/1.128$ is applied. The factors are: Cd and Cd^{113}, 1.3; Xe^{135}, 1.16; Sm, 1.5; Eu, 0.95; Gd, 0.85; Hg, 0.95; U, 0.99 for σ_a and 0.981 for σ_f; U^{235}, 0.981 for σ_a and σ_f. For consistent treatment, the cross sections for Pu^{239} should be multiplied by the factor 1.075, but this factor is highly energy-dependent, increasing to over 2.0 in the range of temperatures used in power reactors. The cross sections of this nuclide (and other fuel nuclides also) are therefore read from curves or tables as a function of energy or temperature.

Energy of Decay. The energy of emitted particles is not given.

Gamma Emission. Decay and neutron capture are usually accompanied by gamma emission, which is not specifically indicated. Also, no notice is taken of exceptions, such as decay unaccompanied by γ rays, or emission, by internal conversion, of orbital electrons instead of γ rays.

Isomeric Transition.† Isomeric states are indicated only where they arise from neutron activation and have a long enough half-life to affect significantly the activity of the irradiated material.

More Than One Mode of Decay. The principal mode is given first; modes accounting for less than 10 per cent of the decay are given in parentheses.

3.22 Important Charts of Nuclides. Charts of the nuclides usually include all known nuclides, stable and unstable, and may give as much nuclear data as consistent with readability.

The General Electric "Chart of the Nuclides" (1956). Copies of this important wall chart plus an accompanying booklet may be obtained free by writing to Dept. 2-119, General Electric Co., Schenectady, N.Y. This chart was used as the primary reference in preparing Table 23.

Trilinear Chart of Nuclides (1957). This is a detailed large-scale accordion-folded strip chart by W. H. Sullivan, for sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C.

4 NUCLEAR DATA USED IN REACTOR CALCULATIONS

See Sec. 2, by H. Soodak, and Sec. 6-2, by J. R. Dietrich. For references to nuclear data presented in this handbook, see Table 5 of this section. See also Ref. 3.

4.1 Cross Sections and Related Data

4.11 Moderators. Nuclear properties of moderators are given in Table 13 of Sec. 2 and Tables 13 and 14 of Sec. 6-2. Table 24 of Sec. 1-1 gives a collection of these properties. For age of metal-water mixtures, see Table 14 of Sec. 6-2.

Heavy Water. The absorption coefficient and diffusion length of heavy water of commercial isotopic purity are greatly decreased by light-water impurity. For heavy water of ordinary purity

$$\Sigma_a \approx 0.000033(1 + 6.56x)$$

$$L^2 \approx 28{,}900\left(\frac{1 - 0.05x}{1 + 6.56x}\right)$$

* Superscript numbers refer to References at end of subsection.

† Isomeric states (excited states of appreciable half-life) are common in nuclides formed by neutron capture; in the majority of cases, the higher isomer reaches the ground state by gamma emission (isomeric transition) with a half-life measured in minutes or less, although sometimes extending to many months.

Table 24. Nuclear Properties of Moderators at 20°C (68°F)

By L. J. Templin *

Moderator	Reference density, g/cm³	Nuclei per cm³	Thermal properties						Epithermal properties						
			$\bar{\Sigma}_a$ (cm⁻¹)	Σ_s (cm⁻¹)	λ_{tr} (cm)	D (cm)	L (cm)	L^2 (cm²)	ξ	Σ_s	$\overline{\xi\Sigma_s}$	λ_{tr}	D	$\sqrt{\tau}$	τ
H_2O	1.00†	0.0334	0.0195	3.51	0.48	0.159	2.85††	8.12††	0.920	1.40‡	1.28‡	2.57	1.13¶	5.6	31
D_2O	1.105	0.0332	0.000033§	0.480	2.86	0.953	170§	28900§	0.509	0.351‡	0.179‡	3.70	1.29¶	1.12	125
Be	1.84	0.1229	0.00109	0.86	1.43	0.481	21.0	441	0.2078	0.737	0.153	1.47	0.562	9.85	97
Graphite	1.60	0.0802	0.000342**	0.385	2.75	0.917	51.8	2680	0.1589	0.385	0.612	2.75	1.016¶	19.08	364

* Collection of these data into a single table has made it necessary in some cases to resolve differences in selections by Soodak (Sec. 2) and Dietrich (Sec. 6-2). Mr. L. J. Templin has kindly undertaken the selection of values in this table.

† Actually 0.998 at 20°C. Calculated quantities are for a reference density of 1 g/cm³.

‡ Values are from about 0.01 Mev to thermal. Σ_s averaged from 2 Mev to thermal is less.

¶ Calculated from the formula

$$D_f = \frac{1}{3}\frac{\displaystyle\int_{u_{th}}^{0.5} \frac{\displaystyle\int_u^{0.5} 15 f(n) e^{-n}\, dn}{\bar{\xi}[\Sigma_s(u)]^2 \overline{(1-\tilde{u}_0)}}\, du}{\displaystyle\int_{u_{th}}^{0.5} \frac{\displaystyle\int_u^{0.5} 15 f(n) e^{-n}\, dn}{\bar{\xi}\Sigma_s(u)}\, du}$$

§ The values are considerably lower for commercial D_2O containing a fraction of a per cent of H_2O (see Art. 4.11).

** Based on commercial graphite having $\sigma_a = 0.0048$ barn.

†† Values of 2.73 and 2.76, and a compromise value of 2.80 are often used for L, with corresponding values of 7.45, 7.62, and 7.84 for L^2.—Ed.

Table 25. Average Absorption and Fission Cross Sections of U^{233}, U^{235}, and Pu^{239}, as a Function of Temperature*

Temperature, °C	U^{233}				U^{235}					Pu^{239}					
	σ_a		σ_f		σ_a		σ_f		η	σ_a		σ_f		η	
	$r = 0$	$r = 0.07$	$r = 0$	$r = 0.07$	$r = 0$	$r = 0.07$	$r = 0$	$r = 0.07$	$r = 0$	$r = 0$	$r = 0.07$	$r = 0$	$r = 0.07$	$r = 0$	$r = 0.07$
20	589.4	629.4	531.4	565.3	680.4	681.9	568.6	566.7	2.076	1103.2	1270.3	776.8	869.4	2.049	1.9915
40	588.9	630.3	531.0	566.0	676.9	678.9	566.1	564.4	2.078	1122.1	1291.9	786.6	880.8	2.040	1.984
60	588.6	631.3	530.7	566.8	673.4	675.7	563.6	562.1	2.079	1143.5	1315.1	797.8	893.2	2.030	1.976
80	588.3	632.3	530.5	567.6	670.1	672.8	561.2	559.9	2.0805	1167.7	1340.4	810.5	906.7	2.020	1.968
100	588.1	633.4	530.3	568.4	667.0	670.1	559.1	557.9	2.082	1194.7	1368.2	824.8	921.6	2.009	1.960
120	587.8	634.3	530.0	569.2	664.2	667.7	556.8	555.8	2.083	1225.0	1398.6	841.0	938.0	1.998	1.952
140	587.8	635.4	530.0	570.1	661.6	665.4	554.8	554.0	2.083	1258.4	1431.5	859.0	956.0	1.986	1.943
160	587.8	636.6	530.0	571.0	659.0	663.0	552.8	552.2	2.084	1295.0	1466.5	878.8	975.2	1.975	1.935
180	587.8	637.7	530.0	571.9	656.7	661.0	550.9	550.5	2.084	1334.8	1503.8	900.4	995.7	1.963	1.927
200	587.9	638.9	530.1	572.9	654.5	659.1	549.2	548.9	2.0845	1378.0	1543.4	924.0	1017.4	1.951	1.918
220	588.1	640.2	530.3	574.0	652.5	657.3	547.6	547.5	2.085	1424.1	1584.9	949.3	1040.3	1.940	1.910
240	588.4	641.5	530.6	575.1	650.6	655.6	546.0	546.0	2.085	1473.2	1628.2	976.4	1064.3	1.929	1.902
260	588.6	642.7	530.8	576.1	648.9	654.2	544.5	544.5	2.084	1525.1	1673.3	1005.0	1089.1	1.918	1.894
280	588.9	643.9	531.0	577.1	647.3	652.8	543.0	543.1	2.084	1579.4	1719.0	1035.2	1114.4	1.907	1.8865
300	589.1	645.1	531.2	578.1	645.8	651.4	541.6	541.8	2.0835	1636.1	1765.2	1066.8	1140.1	1.897	1.879
320	589.4	646.3	531.5	579.1	644.5	650.2	540.4	540.7	2.083	1694.8	1811.7	1099.5	1165.8	1.888	1.8725
340	589.7	647.5	531.7	580.2	643.3	649.1	539.2	539.5	2.0825	1755.5	1858.0	1133.4	1191.4	1.879	1.866
360	590.2	648.9	532.2	581.4	642.1	648.0	538.2	538.5	2.082	1817.7	1903.6	1168.3	1216.7	1.870	1.860
380	590.5	650.1	532.5	582.4	641.0	647.0	537.1	537.4	2.0815	1881.2	1948.4	1204.0	1241.6	1.862	1.854
400	590.9	651.3	532.8	583.4	640.0	646.1	536.1	536.4	2.081	1945.8	1992.4	1240.4	1266.2	1.855	1.849
2,200 m/sec	$\sigma_a = 590.0$		$\sigma_f = 532.0$		$\sigma_a = 697.8$, $\eta = 2.070$		$\sigma_f = 581.6$			$\sigma_a = 1027.28$, $\nu = 2.910$		$\sigma_f = 738.02$, $\eta = 2.0906$			

* From C. H. Westcott: "Effective Cross Section Values for Well-moderated Thermal Reactor Spectra." See Art. 4.12 and Ref. 2.

where x is the weight per cent of light water. Other properties are affected in smaller degree.

4.12 Fuel. In reactor calculations, the average absorption cross section of a $1/v$ absorber is calculated from the 2,200-m/sec reference value by applying factors for the Maxwell distribution and for temperature (See Sec. 6-2 and Art. 7.12 of Sec. 1-1). The marked departure from $1/v$-dependence exhibited by fuel nuclides is most simply allowed for by substituting adjusted cross sections that will give the correct reaction rate when $1/v$-dependence is assumed. Table 25 gives the Westcott[2] adjusted values for U^{233}, U^{235}, and Pu^{239}. The Maxwell distribution factor (0.886 or 1/1.128) and the temperature factor $[\sqrt{293/(273 + t°C)}]$ must still be applied, since Table 25 gives only the secondary corrections necessary to give $1/v$-equivalent cross sections.

$$\bar{\sigma} = \frac{\hat{\sigma}\sqrt{293/(273 + t)}}{1.128}$$

where t = temperature, °C

The parameter r in Table 25 takes into account the density of epithermal neutrons in the $1/E$ "tail" from slowing down of fission neutrons.

$$r = \frac{\text{neutron density in "tail"}}{1.01 \times (\text{total neutron density})}$$

The table assumes the tail to be superimposed on the Maxwell distribution, with a cutoff at $5kT$ $[4.31T(°K) \times 10^{-4}$ ev] as the lower limit. The condition $r = 0$ corresponds to a pure Maxwell distribution, and the values for this condition may be used for calculations of well-moderated reactors. For other reactors, the parameter r must be estimated or calculated from reactor theory, and Table 25 may then be read by interpolation. Westcott states that r for the NRX reactor is 0.03 in the moderator and 0.07 in the fuel.

Since reactor calculations are usually based on the thermal-neutron density and the Westcott cross sections are based on total neutron density, some further adjustment is indicated for consistent treatment of diffusion-dependent terms in reactor theory. The adjustment is, however, dependent on the particular model used in calculation, and, since it is usually not very important, it is generally ignored.

4.13 Fission Products. Table 26 gives absorption cross sections of fission products and yields of xenon and samarium. See Tables 15 to 17 of Sec. 2.

4.14 Other Materials. Table 27 gives data for other common constituents of reactor cores. Table 19 of Sec. 2 gives data for all naturally occurring elements.

Table 26. Fission-product Thermal Cross Sections

Fission product	Yield per fission, per cent	Cross section, barns		Handbook reference
		2,200 m/sec	Maxwell distribution at 20°C	
Xenon^{135},*:				
Direct from fission	0.3			Tables 15 and 16 of Sec. 2. See also Table 4 of Sec. 11.
Indirect	6.6			
Total	6.9	2.72×10^6	2.80×10^6	
Samarium^{149},*	1.3	6.6×10^4	5.0×10^4	Table 16 of Sec. 2. Table 19 of Sec. 2
Other fission products:				
At zero burnup	100	80	71†‡	Art. 4.3 of Sec. 2
After 10 per cent burnup at $\phi = 10^{14}$	100	65	58†‡	

* The tabulated yields (or yields close to them) are commonly used in reactor calculations. Xenon yields given by different observers cover a wide range. The cross section data are from BNL 325, Supplement 1.

Two sets of data for Xe^{135} (S. Bernstein and E. C. Smith), combined with two assumptions of spin factor, give four possible values, ranging from 2.19 to 2.76 megabarns for 2,200-m/sec neutrons. The four values for the average cross section over a Maxwell distribution would, according to Westcott's analysis,[2] range from 2.17 to 2.83 megabarns. Westcott prefers the highest values, which are in close agreement with those of BNL 325. The $\hat{\sigma}$ values for Xe^{135} given by Westcott are:

Temperature, °K	300	400	500	600	700
$\hat{\sigma}$, megabarns	3.208	3.388	3.419	...	3.251

Temperature and Maxwell distribution factors must be applied to $\hat{\sigma}$ for conventional reactor calculations (see Art. 4.12).

† Cross section per fission.

‡ Assuming $1/v$ dependence.

Table 27. Macroscopic Properties

Element and atomic number	General properties				Thermal Microscopic cross sections		
	Atomic weight A, g	Reference density ρ, g/cm³	Atoms per cm³ $N = \frac{0.6023 \times 10^{24}}{A} \rho$ cm⁻²	$1 - \bar{\mu}_0$	σ_a cm²	$\bar{\sigma}_a$ cm²	σ_s cm²
					$\times 10^{-24}$	$\times 10^{-24}$	$\times 10^{-24}$
$_{6}$C	12.01	1.600	.0802	.9440	.0032	.00284	4.8
$_{11}$Na	22.997	0.97	.0254	.9708	.505	.448	4.0
$_{12}$Mg	24.32	1.74	.0431	.9724	.063	.0558	3.6
$_{13}$Al	26.98	2.699	.0603	.9751	.230	.204	1.4
$_{14}$Si	28.09	2.33	.0500	.9761	.13	.115	1.7
$_{15}$P	30.975	1.82	.0354	.9783	.19	.168	5
$_{16}$S	32.066	2.07	.0389	.9790	.49	.434	1.1
$_{19}$K	39.100	0.86	.0132	.9828	1.97	1.75	1.5
$_{22}$Ti	47.90	4.54	.0571	.9860	5.6	4.96	4
$_{24}$Cr	52.01	7.19	.0833	.9871	2.9	2.57	3
$_{25}$Mn	54.93	7.43	.0815	.9878	13.2	11.7	2.3
$_{26}$Fe	55.85	7.87	.0849	.9880	2.53	2.24	11
$_{27}$Co	58.94	8.9	.0909	.9886	37	32.8	7
$_{28}$Ni	58.69	8.9	.0913	.9885	4.6	4.08	17.5
$_{29}$Cu	63.54	8.96	.0849	.9894	3.62	3.27	7.2
$_{40}$Zr	91.22	6.5	.0429	.9926	.180	.160	8
$_{41}$Nb	92.91	8.55	.0554	.9928	1.1	.97	5
$_{42}$Mo	95.95	10.2	.0640	.9930	2.5	2.22	7
$_{50}$Sn	118.7	7.298	.0370	.9943	.60	.532	4
$_{74}$W	183.92	19.3	.0632	.9963	19.2	17.0	5
$_{82}$Pb	207.21	11.34	.0330	.9968	.170	.151	11
$_{83}$Bi	209.00	9.8	.0282	.9968	.032	.0284	9
$_{90}$Th	232.12	11.5	.0298	.9971	7.45	6.2	12.5

* Table 27 gives data for elements likely to be present as important constituents of reactors. Table 24 should be used for

of Elements for Thermal Reactors*

properties				Epithermal properties						
Macroscopic properties				Microscopic properties		Macroscopic properties				Element
$\Sigma_a = N\bar{\sigma}_a$ cm^{-1}	$\Sigma_s = N\sigma_s$ cm^{-1}	$\Sigma_{tr} = \Sigma_s(1 - \bar{\mu}_0)$ cm^{-1}	$\lambda_{tr} = 1/\Sigma_{tr}$ cm	σ_s cm^2	ξ	$\Sigma_s = N\sigma_s$ cm^{-1}	$\xi\Sigma_s$ cm^{-1}	$\Sigma_{tr} = \Sigma_s(1 - \bar{\mu}_0)$ cm^{-1}	$\lambda_{tr} = 1/\Sigma_{tr}$ cm	
				$\times 10^{-24}$						
.000228	.385	.364	2.75	4.8	.1589	.385	.061	.364	2.75	C
.0114	.102	.099	10.1	3.1	.0852	.079	.0067	.076	13.1	Na
.00241	.155	.151	6.6	3.4	.0807	.147	.0118	.142	7.0	Mg
.0123	.084	.082	12.2	1.4	.0730	.084	.0062	.082	12.2	Al
.0058	.085	.083	12.1	2.2	.0702	.110	.0077	.107	9.3	Si
.0060	.18	.17	5.8	3.4	.0637	.120	.0077	.118	8.5	P
.0169	.043	.042	24	1.1	.0616	.043	.0026	.042	24	S
.0231	.0199	.0195	51	2.1	.0507	.028	.00141	.027	37	K
.283	.23	.23	4.4	4.2	.0415	.240	.0100	.240	4.23	Ti
.214	.25	.25	4.1	3.9	.0383	.325	.0124	.321	3.12	Cr
.953	.187	.185	5.4	1.9	.0363	.155	.0056	.153	6.5	Mn
.190	.93	.92	1.08	11.4	.0357	.968	.0345	.956	1.05	Fe
2.98	.64	.63	1.6	5.8	.0338	.528	.0178	.521	1.92	Co
.372	1.60	1.58	.633	17.4	.0340	1.59	.0540	1.57	.637	Ni
.273	.612	.605	1.65	7.7	.0314	.654	.0205	.647	1.55	Cu
.00685	.34	.34	2.9	6.2	.0220	.266	.00585	.264	3.79	Zr
.054	.28	.28	3.6	6.5	.0216	.360	.00778	.358	2.80	Nb
.142	.45	.45	2.2	6.0	.0209	.384	.00803	.381	2.62	Mo
.0197	.15	.15	6.8	4.8	.0169	.178	.00300	.177	5.66	Sn
1.08	.32	.31	3.2	5.6	.0109	.354	.00386	.353	2.84	W
.00497	.36	.36	2.8	11.3	.0097	.372	.00361	.371	2.69	Pb
.00080	.25	.25	3.9	9.28	.0096	.262	.00252	.261	3.83	Bi
.222	.373	.372	2.69	12.5	.0087	.373	.00324	.372	2.69	Th

moderators. Table 18 of Sec. 2 gives the basic data for all the elements, and Table 20 of Sec. 2 gives resonance integrals.

4.2 Fission Data

4.21 Neutron Regeneration. Table 28 gives neutron-regeneration data for fuels. Additional data are given in Table 6 of Sec. 2.

$$\alpha = \frac{\nu}{\eta} - 1 \qquad \eta = \nu/(1 + \alpha)$$

4.22 Other Fission Data. References to other data (including fission-neutron spectrum, energy dependence of nuclear properties, spontaneous fission rates, photoneutron yield, fission-product distribution, and delayed-neutron data) are given in Table 5 of this section.

Table 28. Neutron-regeneration Data*

	U^{233}	U^{235}	Pu^{239}†	Natural uranium
Neutrons per thermal fission ν	2.52	2.47	2.91	2.47
Neutrons per thermal absorption η	2.28	2.07	2.09	1.34
Captures per fission $\sigma_c/\sigma_f = \alpha = \frac{\nu}{\eta} - 1$	0.105	0.192	0.39	0.85

* "World consistent" data for 2,200-m/sec neutrons, Ref. 1, 1957 Supplement. These data differ slightly from those of Sec. 2, Table 6. For the neutron spectrum of a thermal reactor, α and η can be calculated from Table 25 and ν of this table.

† All values in the table are for 2,200-m/sec neutrons. In thermal reactors, the strong resonance of Pu^{239} causes α to be considerably higher and η considerably lower than the tabulated values. See Table 25 of this section and Art. 6.5 of Sec. 6-1.

5 CONSTANTS AND FORMULAS RELATING POWER, FLUX, AND FUEL CONSUMPTION

See Sec. 6-2, by J. R. Dietrich, and Sec. 12-2, by L. E. Link and Walter H. Zinn.

5.1 Energy from Fission

The available energy per fission is usually taken as 200 Mev.* The energy distribution for fission of U^{235} by thermal neutrons is given in Table 1 of Sec. 2. Table 29 gives some frequently used energy and fission equivalents. See also Table 28 of Sec. 1-3.

Table 29. Fission Energy Equivalents*

1 fission	$= 3.20 \times 10^{-11}$ joules (or watt-sec)
1 joule	$= 3.12 \times 10^{10}$ fissions
1 watt	$= 3.12 \times 10^{10}$ fissions/sec
1 Mwd (megawatt-day)	$= 2.70 \times 10^{21}$ fissions

* Based on 200 Mev per fission and 1 ev = 1.60203×10^{-12} ergs.

5.2 Flux-Power Relations

5.21 Average Thermal Flux and Power

$$\bar{\phi}_{th} = K \times 10^{13} \times \text{specific power}$$

$$\text{Mw} = \frac{1}{K} \times \frac{\bar{\phi}_{th}}{10^{13}} \times \text{kg}$$

where $\bar{\phi}_{th}$ = average thermal flux,† $n/(\text{cm}^2)(\text{sec})$
Mw = power, Mw

* Formulas in this article are based on 200 Mev/fission for the total heat generation in the reactor and shielding. A small adjustment must be made to apply the formulas to heat generation in the core alone.

† $\bar{\phi}_{th}$ is the average thermal flux in fuel. For an effectively homogeneous reactor this is the same as the average over the core.

kg = mass of fissionable material, kg
specific power = Mw/kg of fissionable nuclide
K = constant as given below

General Formula for the Constant K

$$K = 1{,}036\left(\frac{A}{\bar{\sigma}_f E}\right) \qquad \text{Eq. (96) of Sec. 6-2}$$

where A = atomic weight of isotope, g
$\bar{\sigma}_f$ = microscopic fission cross section (averaged over the Maxwell distribution at operating temperature, with allowance for deviation from $1/v$ dependence), barns
E = energy per fission, Mev

$$E = (8.617 \times 10^{-5})T(°\text{K}) = (4.787 \times 10^{-5})T(°\text{R})$$

For a Mixture of Fuels

$$\frac{1}{K} = \sum_i \frac{1}{K_i} \qquad K = 1\Big/\sum_i \frac{1}{K_i}$$

Value of K and 1/K for U^{235}. Table 30 gives K and $1/K$ for U^{235} based on A = 235 g, E = 200 Mev, $\bar{\sigma}_f$ ($r = 0$) from Table 25.

Table 30. K and $1/K$ for U^{235}

	Temperature, °C								
	20	50	100	150	200	250	300	350	400
K	2.41	2.55	2.77	2.98	3.18	3.36	3.54	3.72	3.88
$1/K$	0.414	0.392	0.361	0.335	0.315	0.297	0.282	0.269	0.257

5.22 Fast Flux. *Virgin Flux*

$$\phi_v = \frac{3.12 \times 10^{10}\ \nu \times (\text{watts/cm}^3)}{\Sigma}$$

where ν = number of neutrons per fission (2.47 for U^{235})
Σ = total macroscopic cross section of reactor material for virgin neutrons

Two-group Fast Flux. See Art. 6.5 of Sec. 6-2 (also Art. 7.71 of this section).

5.3 Consumption of Fissionable Material

See Art. 1.1 of Sec. 12-2.

One gram of fuel *fissioned* yields approximately 0.95 Mwd. Table 31 gives fuel consumption (including nonfission capture) based on 200 Mev per fission.

Table 31. Consumption Rate of Fuel, Grams per Megawatt-day*

	General formula (A = atomic weight of nuclide, $\alpha = \sigma_c/\sigma_f$)	Thermal reactors		
		U^{233} $\alpha = 0.105$	U^{235} $\alpha = 0.192$	Pu^{239} $\alpha = 0.39$
Consumption, grams/Mwd.....	$0.00445A(1 + \alpha)/\epsilon$	$1.15/\epsilon$	$1.25/\epsilon$	$1.48/\epsilon$

* For highly enriched reactors $\epsilon = 1$; for large natural-uranium graphite reactors $\epsilon \approx 1.04$.

5.31 Exposure and Burnup Units. Based on the rough approximation that 1,000 kw of fission heat corresponds to fission of 1 g of fuel per day:

$$1 \text{ per cent of fuel fissioned} \approx 10{,}000 \text{ Mwd/tonne}$$

More exactly:

$$\begin{aligned} 1 \text{ per cent of fuel fissioned} &= 9{,}500 \text{ Mwd/tonne} \\ &= 8{,}600 \text{ Mwd/ton (2,000 lb)} \end{aligned}$$

For 1 per cent of atoms consumed, including radiative capture, divide the 9,500 or 8,600 by $(1 + \alpha)$. For nominal values of α (Table 31), the Mwd/tonne equivalents of 1 per cent burnup of U^{233}, U^{235}, Pu^{239} are 8,600, 8,000, and 6,800, respectively.

For average burnup based on heat developed in the core, multiply these numbers by

$$\text{Heat retained in core/(heat in core + heat outside core)}$$

5.4 Conversion and Breeding

5.41 Available Neutrons. The parameters η, f, p, B^2, and τ have their customary meanings (see Art. 7 of this section). Values of η at 2,200 m/sec: U^{233}, 2.28; U^{235}, 2.07; Pu^{239}, 2.09.

$$\frac{\text{Neutrons available for breeding or converting in core}}{\text{Neutrons absorbed in fissionable material}} = \eta(1 - L) - \frac{1}{f} = \eta P - \frac{1}{f}$$

where L = leakage
P = probability that a neutron will not leak from the core (see Art. 2.31 of Sec. 12-2)

5.42 Conversion Ratio

$$C = \eta - 1 - l$$

where C = conversion ratio = $\dfrac{\text{atoms generated}}{\text{atoms destroyed}}$
l = neutrons lost (by leakage and capture in all but fissionable materials) per neutron absorbed in fissionable material (see Art. 2.33 of Sec. 12-2)

Initial Conversion Ratio

$$C_{initial} = \frac{\sigma_a{}^{U^{238}}}{r\sigma_a{}^{U^{235}}} + \eta^{U^{235}}\epsilon(1 - p)e^{-B^2\tau}$$

where r = fraction of U^{235} in uranium mixture (see Art. 2.33 of Sec. 12-2)

5.43 Breeding or Conversion Gain ($C > 1$)

$$G = C - 1 = \eta - 2 - l \qquad \text{(see Art. 2.34 of Sec. 12-2)}$$

5.44 Doubling Time

$$\text{Doubling time in days} = \frac{\text{inventory of fissionable material, grams}}{G \text{ (daily fuel consumption, grams)}}$$

The daily fuel consumption is obtained from Table 31 and the megawatts of heat.

5.45 Fuel Utilization ($C < 1$)

$$\text{Upper limit} = \frac{R}{1 - C}$$

where R = fraction of fissionable material in mixture of fuel and fertile material (see Art. 2.34 of Sec. 12-2)

6 NUMERICAL INTEGRATION BY SIMPSON'S RULE

Various methods of numerical integration, including Simpson's rule, are discussed in Art. 1.4 of Sec. 3-2. Simpson's rule is widely used for integrating nonanalytic

functions, including determination of reactor constants from empirical data. Practical application is summarized below.

To evaluate $\int_a^b y\,dx$, where y is a function of x:

1. Divide the interval ab into an even number of equal intervals h. If n is the number of intervals, $h = (b - a)/n$.
2. Evaluate y (analytically, from tables, by measurement, or from empirical data) at $x = a$, $x = a + h$, $x = a + 2h$, . . . , $x = b$.
3. $\int_a^b y\,dx = \frac{h}{3}[y_a + 4y_{a+h} + 2y_{a+2h} + 4y_{a+3h} + 2y_{a+4h} + \cdots + 4y_{b-h} + y_b]$

Example: To evaluate $\int_1^2 xJ_0(x)\,dx$ by Simpson's rule, divide the interval 1 to 2 into four equal parts, $h = 0.25$

x	1	1.25	1.5	1.75	2.0
From tables, $J_0(x)$	0.7652	0.6459	0.5118	0.3690	0.2239
$xJ_0(x)$	0.7652	0.8074	0.7677	0.6457	0.4478

$$\int_1^2 xJ_0(x)\,dx = \frac{0.25}{3}[0.7652 + 3.2294 + 1.5354 + 2.5828 + 0.4478]$$
$$= 0.7134$$

The correct solution, using the analytic relation $\int xJ_0(x)\,dx = xJ_1(x)$, is 0.7133.

7 REACTOR CALCULATIONS

See Sec. 6-2, by J. R. Dietrich.

This article, like others of Sec. 1-1, is a guide to a major section of the handbook and should not be used alone. Formulas have been selected from Sec. 6-2 to highlight the steps of criticality calculation and to serve as a framework for references to the detailed instructions contained in Sec. 6-2. For additional tabulated data, see also Table 5 of Sec. 1-1. Article 11 of Sec. 6-2 gives examples showing application of formulas.

Nomenclature

A = atomic weight of element, g. Mass number of nucleus relative to mass of neutron, dimensionless
B^2 = buckling, cm^{-2}
D = diffusion coefficient, cm.
E = neutron energy, ev. Also a dimensionless constant
F = dimensionless constant
f = thermal utilization, dimensionless
k = multiplication constant in infinite medium, dimensionless
k(in kT) = Boltzmann constant, ev/deg
L = diffusion length, cm
L_f = slowing-down length, cm
M^2 = migration area, cm^2
N = number of nuclei per cubic centimeter, cm^{-3}
N = 0.6023×10^{24} = Avogadro's number, atoms per gram atomic weight, chemical scale, g^{-1}
p = resonance escape probability, dimensionless
T = absolute temperature, °K or °R
t = temperature, °C or °F
V = volume, cm^3
v = neutron velocity, cm/sec
ϵ = fast fission factor, dimensionless
η = regeneration factor, dimensionless
κ = reciprocal of diffusion length, cm^{-1}

λ = mean free path of neutron, cm
$\bar{\mu}_0$ = average cosine of scattering angle (numerically $\bar{\mu}_0 = 2/3A$), dimensionless
ν = average number of neutrons emitted per fission, dimensionless
ξ = average logarithmic change of energy per collision, dimensionless
ρ = density, g/cm^3
Σ = macroscopic cross section, cm^{-1}
σ = microscopic cross section, cm^2
τ = Fermi age, cm^2
ϕ = neutron flux, cm^{-2} sec^{-1}

Subscripts

0 = initial or standard condition
1 = fuel region
2 = moderator region
3 = clad region
a = absorption
f = fast (when used with σ denotes fission)
i = ith component of a mixture
s = scattering
th = thermal
tr = transport

7.1 Average Thermal Microscopic Absorption Cross Section $\bar{\sigma}_a$

7.11 Energy of Thermal Neutrons. The reference energy used in selection of cross-section data is the energy corresponding to the most probable velocity in the Maxwell distribution at the operating temperature [see Eq. (63) of Sec. 6-2].

$$kT = 8.617 \times 10^{-5}T \quad \text{ev} \qquad T = {}^\circ\text{K}$$
$$= 4.787 \times 10^{-5}T \quad \text{ev} \qquad T = {}^\circ\text{R}$$

Effective Neutron Temperature T. In thermal reactors, T may exceed the moderater temperature by about 50°C. See Art. 4.4 of Sec. 6-2 and formula in Art. 5.3 of Sec. 2.

7.12 $1/v$ Absorbers with Maxwell Distribution. See Eqs. (65) and (66) of Sec. 6-2.

From 2,200-m/sec Data

$$\bar{\sigma}_a = \frac{\sigma_{a0}}{1.128}\sqrt{\frac{293}{(t + 273)}} \qquad t = {}^\circ\text{C}$$
$$= \frac{\sigma_{a0}}{1.128}\sqrt{\frac{529}{(t + 460)}} \qquad t = {}^\circ\text{F}$$
$$= \frac{\sigma_{a0}}{1.128}\sqrt{\frac{0.0253}{E}} \qquad E = \text{ev}$$

where σ_{a0} = absorption cross section at standard (monoenergetic) neutron velocity of 2,200 m/sec (corresponding to an energy of approximately 0.0253 ev and a kT temperature of approximately 20.4°C or 68.8°F)
$\bar{\sigma}_a$ = average cross section to be applied in conjunction with total thermal flux

The number $2/\sqrt{\pi} = 1.128$ is the correction for Maxwell distribution. The square-root factor in the formulas is the temperature correction for reactors in which the neutron temperature is not 20°C (69°F).

Table 19 of Sec. 2 gives σ_{a0} for elements and nuclides, and Table 27 of this section gives $\bar{\sigma}_{a0} = \sigma_a/1.128$ for some important elements.

From Curves of σ_a as a Function of the Energy kT: For curves of σ_a for monoenergetic neutrons of energy kT, see "Neutron Cross Sections."[1]

$$\bar{\sigma}_a = \sigma_{a(kT)}/1.128$$

7.13 Non-$1/v$ Absorbers with Maxwell Distribution. See Art. 4.3 of Sec. 6-2. The temperature-correction factor and the 1.128 distribution factor used for $1/v$

absorbers are not applicable to non-$1/v$ absorbers. In general, from Eq. (66) of Sec. 6-2,

$$\bar{\sigma}_a = \int_{E_1}^{E_2} \sigma_a E \exp(-E/kT)\, dE / \{kT[(E_1 + kT)\exp(-E_1/kT) - (E_2 + kT)\exp(-E_2/kT)]\}$$

where the integral can be evaluated (over a sufficient range E_1 to E_2) from Ref. 1, using, for example, Simpson's rule (see Art. 6). For practical application the following simpler processes are generally adequate.

U^{233}, U^{235}, *and* Pu^{239} (*non-*$1/v$ *absorbers*)

Determine $\bar{\sigma}_a$ or $\bar{\sigma}_f$ for U^{233}, U^{235}, or Pu^{239} from δ in Table 25 of Sec. 1-1.

Other non-$1/v$ *Absorbers*

1. For a temperature of 20°C, the cross section is read from Table 18 or 19 of Sec. 2, the 1/1.128 factor is applied, and the result is multiplied by the correction factor given in the table (Cd, 1.3; Xe^{135}, 1.16; Sm, 1.5; Eu, 0.95; Gd, 0.85; Hg, 0.95).

2. For temperatures up to about 250°F (120°C), the cross section for the corresponding kT energy may be read from curves[1] and the factors used in (a) may be applied. At still higher temperatures the correction may be increasingly in error.

3. Westcott[2] gives δ_a for Mn, Co, Rh, In^{115}, Gd, and Au, in addition to the fuel and fission product nuclides discussed in Arts. 4.12 and 4.13.

7.14 The $1/E$ "Tail." The departure from Maxwell distribution at the high-energy end of the spectrum, due to slowing-down neutrons, is discussed in Art. 4.4 of Sec. 6-2; it can generally be ignored (Table 25 and other data from Westcott[2] take the $1/E$ "tail" into account).

7.2 Macroscopic Cross Sections Σ

7.21 Single Nuclide or Element (All Cross Sections)

$$\Sigma = N\sigma = \frac{\rho}{A} N\sigma$$

If Σ_0 is known for a material at standard density ρ_0 (e.g., Table 27 of Sec. 1-1 and Table 18 of Sec. 2) Σ at some other density ρ is given by $\Sigma = \Sigma_0(\rho/\rho_0)$.

7.22 Homogeneous Mixtures (All Cross Sections)

$$\Sigma = \sum_i N_i\sigma_i = N \sum_i \frac{\rho_i}{A_i}\sigma_i \qquad \text{Eq. (3) of Sec. 6-2}$$

where ρ_i is the partial density or g/cm³ of the ith material.

7.23 Mean Free Path λ. Single Nuclides or Homogeneous Mixtures (All Cross Sections): $\lambda = 1/\Sigma$.

7.24 Average Macroscopic Absorption Cross Section. Σ_a must be averaged over the Maxwell neutron energy distribution and corrected for temperature. The corrections described for microscopic absorption cross sections may be applied either to σ_a or to Σ_a. Table 27 of Section 1-1 gives Σ_a already corrected for the Maxwell distribution at room temperature; for other temperatures, the temperature correction must be applied.

7.25 Special Cases. Other macroscopic cross sections are discussed in Art. 7.5.

7.3 Slowing-down and Scattering Constants

7.31 Average Logarithmic Change of Energy per Collision. See Art. 3.1 of Sec. 6-2.

$$\xi = \ln\frac{E_0}{E} = 1 + \frac{r}{1-r}\ln r \qquad \text{Eq. (36) of Sec. 6-2}$$

where

$$r = \left(\frac{A-1}{A+1}\right)^2 \qquad \text{Eq. (35) of Sec. 6-2}$$

Values of ξ for the elements are given in Table 18 of Sec. 2. For a mixture of nuclides,

$$\bar{\xi} = \frac{\sum_i \Sigma_{si}\xi_i}{\sum_i \Sigma_{si}} \qquad \text{Eq. (37) of Sec. 6-2}$$

7.32 Average Cosine of Scattering Angle. For isotropic scattering in the center-of-gravity system,

$$\bar{\mu}_0 = \overline{\cos\theta} = 2/(3A) \qquad \text{Art. 2.13 of Sec. 6-2}$$

where θ is the scattering angle in the laboratory system. $1 - \bar{\mu}_0$ is given as $1 - \overline{\cos\theta}$ in Table 18 of Sec. 2.

7.4 Infinite Multiplication Constant*

$$k = \eta\epsilon pf \qquad \text{Eq. (168) of Sec. 6-2}$$

For calculation of p and f in fuel-moderator lattices, the unit cell is idealized (usually to cylindrical geometry), maintaining true fuel and moderator volumes (see Arts. 9.3 and 9.4 of Sec. 6-2).

7.41 Regeneration Factor η. See Art. 9.2 of Sec. 6-2.

If applied to the fissionable isotope,

$$\eta = \nu(\sigma_f/\sigma_a)$$

If applied to a homogeneous fuel mixture

$$\eta = \sum_i \nu_i N_i \sigma_{fi} \Big/ \sum_i N_i \sigma_{ai} \qquad \text{Eq. (169) of Sec. 6-2}$$

Table 28 of Sec. 1-1 gives values of ν and η. For a heterogeneous fuel element, see Eq. (170) of Sec. 6-2. See Art. 11.21 of Sec. 6-2 for a numerical example.

7.42 Thermal Utilization f

$$f = \text{absorption in fuel/total absorption}$$

Homogeneous Mixture

$$f = \frac{\Sigma_{a1}}{\Sigma_{a(\text{total})}} = \frac{\Sigma_{a1}}{\Sigma_{a1} + \Sigma_{a(\text{other})}} = \frac{1}{1 + \dfrac{\Sigma_{a(\text{other})}}{\Sigma_{a1}}} \qquad \text{Eq. (171) of Sec. 6-2}$$

The subscript "other" means materials other than fuel.

Fuel-Moderator Lattice

$$\frac{1}{f} = 1 + \left[\frac{V_2\Sigma_{a2} + V_3\Sigma_{a3}}{V_1\Sigma_{a1}}\right]F + (E - 1) \qquad \text{Eq. (190) of Sec. 6-2}$$

E and F are calculated from Table 8 of Sec. 6-2, and κ_1 and κ_2 (equals $1/L_1$ and $1/L_2$) from Table 13 of Sec. 6-2.

$$\text{Disadvantage factor} = \frac{\bar{\phi}_2}{\bar{\phi}_1} = \frac{\Sigma_{a1}V_1}{\Sigma_{a2}V_2}\left(\frac{1}{f} - 1\right) \qquad \text{Eq. (177) of Sec. 6-2}$$

See Art. 11.22 of Sec. 6-2 for numerical example.

* The macroscopic cross sections entering into reactor theory are usually average values for mixtures of materials over particular flux distributions. Where confusion is unlikely, these average cross sections are represented by Σ instead of $\bar{\Sigma}$.

7.43 Resonance Escape Probability p. See Art. 9.4 of Sec. 6-2.

Homogeneous Mixture

$$p = \exp\left(-\frac{1}{\bar{\xi}}\frac{N_a}{\Sigma_s}I\right) \quad \text{Eqs. (57) and (192) of Sec. 6-2}$$

where $\bar{\xi} = \sum_i \Sigma_{si}\xi_i \Big/ \sum_i \Sigma_{si}$, from Eq. (37) of Sec. 6-2, is calculated from ξ and $\sigma_{s(th)}$ in Table 22 of Sec. 2

$$\frac{N_a}{\Sigma_s} = \frac{\text{number of absorber atoms/cm}^3}{\text{total macroscopic scattering cross section of mixture}}$$

I = resonance integral from Fig. 14 (see also Arts. 3.51 and 3.53 of Sec. 6-2). Article 6 of Sec. 2 also gives absorption integrals

Fuel-Moderator Lattice

$$p = \exp\left[-\frac{1}{\dfrac{V_2\bar{\xi}_2\Sigma_{s2}F}{N_aV_1(A+\mu S/M)} + (E-1)}\right] \quad \text{Eqs. (206) and (200) of Sec. 6-2}$$

If region 2 is a pure moderator,

$$p = \exp\left[-\frac{1}{\dfrac{N_2V_2(\xi_2\sigma_{s2})F}{N_aV_1(A+\mu S/M)} + (E-1)}\right]$$

where N = atoms of resonance absorber (U^{238}) per cubic centimeter of fuel; E and F are calculated from Table 8 of Sec. 6-2 using $\varkappa_1$ and $\varkappa_2$ from Tables 10 and 11 respectively; S/M = (surface/mass) of fuel lump; A and μ are found from Table 9 of Sec. 6-2 (for natural uranium $A = 9.25$, $\mu = 24.7$).

Modified parameters for uranium and thorium, and their extension to diluted fuel lumps, are given in Tables 21 and 22 of Sec. 2 and are discussed in Art. 6.2 of that section.

$\bar{\xi}_{s2}\sigma_2$ for a pure moderator is found from Table 11 of Sec. 6-2 (H_2O, 38.5; D_2O, 5.28; Be, 1.26; C, 0.76). See Art. 11.23 of Sec. 6-2 for a numerical example.

7.44 Fast-fission Effect ϵ

$$\epsilon = 1 + \frac{(\nu - 1 - \Sigma_c/\Sigma_f)\dfrac{\Sigma_f}{\Sigma_t}P}{1 - \left(\dfrac{\nu\Sigma_f + \Sigma_{cl}}{\Sigma_t}\right)P'} \quad \text{Eq. (207) of Sec. 6-2}$$

where cross sections refer to fuel and are defined on p. 6–84. For natural uranium $\epsilon \approx 1 + 0.09P/(1 - 0.52P')$ (see Table 12 of Sec. 6-2).

P and P' are found from Fig. 15 of Sec. 6-2 and $\varkappa_1$ (fuel, thermal) $= 1/L_1$ from Table 13 of Sec. 6-2. P' corresponds to $\varkappa_1/\Sigma_t = 0$. For close-packed lattices of slightly enriched uranium, see Table 12 of Sec. 6-2. See Art. 11.24 of Sec. 6-2 for numerical examples.

7.5 Other Material Constants of Reactors

7.51 Transport Mean Free Path and Transport Cross Section

$$\lambda_{tr} = 1/\Sigma_{tr}$$

Experimental Values. For experimental values of λ_{tr} for moderators, see Table 13 of Sec. 6-2 and Table 24 of Sec. 1-1 (H_2O, 0.48; D_2O, 2.86; Be, 1.43; graphite, 2.75).

Calculated Values. If experimental data are lacking use

$$\Sigma_{tr} = \Sigma_s(1 - \bar{\mu}_0) = \Sigma_s[1 - (2/3A)]$$

Values for important elements are given in Table 27 of Sec. 1-1.

Homogeneous Mixture

$$\Sigma_{tr} = \sum_i \Sigma_{tr,i}$$

Experimental values of $\Sigma_{tr,i}$ should be used wherever available: if ρ_{0i} is the standard density of the ith constituent and ρ_i is grams of material in 1 cm^3, $\Sigma_{tr,i} = \Sigma_{tr,0i}\dfrac{\rho_i}{\rho_{0i}} = \dfrac{1}{\lambda_{tr,0i}}\dfrac{\rho_i}{\rho_{0i}}$ where λ_{tr} can be taken from Table 13 of Sec. 6-2 or Table 24 of Sec. 1-1. For constituents of unknown transport cross section, use $\Sigma_{tr,i} = \Sigma_{si}(1 - \bar{\mu}_{0i})$.

7.52 Thermal Diffusion Coefficient D_{th}. *Homogeneous Medium, Weak Absorber, Experimental Values.* For a pure medium:

$$D_{th} = \frac{\lambda_{tr}}{3} = \frac{1}{3\Sigma_{tr}} = \frac{1}{3\Sigma_s(1 - \bar{\mu}_0)} \qquad \text{Eq. (10) of Sec. 6-2}$$

For moderators λ_{tr} is given in Table 13 of Sec. 6-2; $\lambda_{tr,th}$ and D_{th} in Table 24 of Sec. 1-1.

For a mixture:

$$\lambda_{tr} = 1/\Sigma_{tr}$$

Homogeneous Medium, General Case, Calculated

$$D_{th} = 1\Big/\left[3\Sigma(1 - \bar{\mu}_0)\left(1 - \frac{4}{5}\frac{\Sigma_a}{\Sigma} + \frac{\Sigma_a}{\Sigma}\cdot\frac{\bar{\mu}_0}{1 - \bar{\mu}_0} + \cdots\right)\right] \qquad \text{Eq. (9) of Sec. 6-2}$$

where Σ and Σ_a are total and absorption macroscopic cross sections, respectively.

Fuel-Moderator Lattice. If the ratio of moderator volume to fuel volume is large, use D_{th} of the moderator. Otherwise,

$$D_{th} \approx \frac{1}{3\Sigma_{tr2}}\left[\frac{1 + m}{1 + m(\Sigma_{tr1}/\Sigma_{tr2})}\right]$$

where $m = [f/(1 - f)](\Sigma_{a2}/\Sigma_{a1})$ (see Art. 9.6 of Sec. 6.2)

7.53 Fast Diffusion Coefficient D_f. *Homogeneous Medium.* If an average value of σ_s can be chosen for the moderator constituents over the energy range from fission to thermal

$$\Sigma_{tr} = \sum_i \Sigma_{si}(1 - \bar{\mu}_{0i}) = \sum_i \Sigma_{si}\left(1 - \frac{2}{3A}\right)$$

$$D_f = 1/3\Sigma_{tr}$$

To evaluate for variation of σ_s with E, see Art. 9.8 of Sec. 6-2.

Fuel-Moderator Lattice. Lumping can usually be neglected; treat as homogeneous medium.

7.54 Thermal Diffusion Area

$$L^2 = 1/\kappa^2 = D_{th}/\Sigma_{a,th} \qquad \text{Eq. (208) of Sec. 6-2}$$

Experimental Values for Moderators. L is given in Table 13 of Sec. 6-2, Table 24 of Sec. 1-1, and Table 13 of Sec. 2 (H_2O, 2.85; pure D_2O, 170; Be, 21.0; graphite 52).

Homogeneous Mixture. Calculate from above formula. Use experimental values of D_{th} if available; otherwise calculate from

$$L^2 = \tfrac{1}{3}\lambda_{tr}\lambda_a = 1/(3\Sigma_{tr}\Sigma_a)$$

Moderator-Fuel Lattice. By combination of Eqs. (177), (208), and (209) of Sec. 6-2

$$L^2 = D_{th}\left(1 - f + \frac{\Sigma_{a2}}{\Sigma_{a1}} f\right) \Big/ \Sigma_{a2}$$

If $\Sigma_{a2} \ll \Sigma_{a1}$ $L^2 = L_2^2(1 - f)$ [Eq. (211) of Sec. 6-2], where L_2 (moderator) is found from Table 13 of Sec. 6-2 or an equivalent source.

7.55 Age τ. *Experimental Values for Moderators.* τ is given in Table 14 of Sec. 6-2, Table 24 of Sec. 1-1, and Table 13 of Sec. 2 (H_2O, 31; D_2O, 125; Be, 97; graphite, 364).

Calculated Values for Moderators of Higher Atomic Weight than Beryllium

$$\tau(E) = \int_E^{E_0} \frac{D(E)}{\xi\Sigma_s(E)} \frac{dE}{E} \qquad \text{Eq. (42) of Sec. 6-2}$$

Effective age $\bar{\tau}$ from fission source: See Art. 9.9 and Eq. (48) of Sec. 6-2.

Moderator-Fuel Lattices. Use τ for moderator if fuel is uranium metal and (volume fuel)/(volume moderator) is small. (See Art. 9.9 of Sec. 6-2).

7.56 Two-group Slowing-down Area

$$L_f^2 = \frac{1}{\varkappa_f^2} \qquad \text{Eq. (84) of Sec. 6-2}$$

For hydrogen-moderated reactors,

$$L_f^2 = \tau \qquad \text{Art. 5.6 of Sec. 6-2}$$

For other moderators: for example,

$$L_f^2 = (e^{\tau B^2} - 1)/B^2 \quad \text{(see Arts. 5.6 and 9.9 of Sec. 6-2)}$$

where B^2 is the buckling according to Fermi theory obtained either from the characteristic equation $(1 + L^2B^2)e^{\tau B^2} = k$ (if the core material has been selected) or from the equivalent bare-reactor equation (Table 5 of Sec. 6-2), if the geometry has been chosen.

7.57 Fictitious Fast Absorption Cross Section Σ_{af}

$$\Sigma_{af} = D_f/L_f^2$$

7.58 Extrapolation Distance

For plane black boundary

$$\epsilon = 0.71\lambda_{tr}$$

For other cases see Art. 2.22 of Sec. 6-2.

7.6 Characteristic Equations

Fermi Age Theory

$$(1 + L^2B^2)e^{\tau B^2} = k \qquad \text{Eq. (82) of Sec. 6-2}$$

Modified One-group Theory

$$1 + M^2B^2 = k \qquad B^2 = \frac{k - 1}{M^2} \qquad \text{Eq. (93) of Sec. 6-2}$$

where $M^2 = L^2 + L_f^2$

Two-group Theory

$$(1 + L^2B^2)(1 + L_f^2B^2) = k \qquad \text{Eq. (89) of Sec. 6-2}$$

Solutions of Eq. (89) for core and reflector are given in Eqs. (109) to (112) of Sec. 6-2. $B^2 = (k - 1)/M^2$ is, in all cases, a first approximation of the fundamental buckling, where M^2 is $L^2 + \tau$(Fermi) or $L^2 + L_f^2$ (two-group).

7.7 Solution of Wave Equations for Critical Reactors

$$\nabla^2\phi + B^2\phi = 0 \qquad \nabla^2\phi - \varkappa^2\phi = 0 \quad \text{Eqs. (76) and (13) of Sec. 6-2}$$

7.71 Bare Reactor. Formulas for flux and critical dimensions are given in Table 5 of Sec. 6-2.

Two-group Fast Flux

$$\frac{\phi_f}{\phi_{th}} = \frac{\Sigma_{a,th} + D_{th}B^2}{p\Sigma_{af}} = \frac{k}{p}\frac{\Sigma_{a,th}}{\Sigma_{af}}\frac{1}{1 + L_f^2B^2}$$

$$\phi_f = \frac{\nu \times \text{fissions}/(\text{cm}^3)(\text{sec})}{\Sigma_{af}(1 + L_f^2B^2)}$$

Eqs. (92), (208), and (89) of Sec. 6-2

Leakage formulas are given in Art. 6.6 of Sec. 6-2.

7.72 Reflected Reactors, Two-region, Two-group. *Coupling Coefficients*

$$S = \frac{\phi_f}{\phi_{th}} = \frac{\Sigma_{a,th} + D_{th}B^2}{p\Sigma_{af}}$$

Eq. (92) of Sec. 6-2

Other coupling coefficients are given in Eqs. (127) to (130) of Sec. 6-2.

Solutions. Solutions are given in Sec. 6-2 as follows:

Cylinder:

Radial reflector	Art. 7-1
End reflector	Art. 7-2
Rectangular parallelepiped reflected on one pair of faces	Art. 7-3
Sphere	Art. 7-4
Bare cylindrical reactor with central region	Art. 7-5
Reflectors in more than one direction: reflector-savings method	Art. 7-7

See also example of complete reactor solution in Art. 11.

Solution by Matrix Method (for two or more regions). See Art. 8 and example in Art. 11.7 of Sec. 6-2.

8 CALCULATION OF NUCLEAR RADIATION

See Sec. 7-1, by J. M. West.

Nomenclature

A = activity, disintegrations per second. For reactor coolant, A = disintegrations per second per cubic centimeter of coolant

M_1 = atomic mass of original nuclide, M_2 = atomic mass of daughter or product nuclide

N = number of atoms of nuclide at time t. For reactor coolant, N = number, at time t, per cm³ of coolant. N_1 refers to original nuclide, N_2 to daughter or product nuclide, N_0 (abbreviation of N_{10}) = number of original atoms at time $t = 0$

T = total operating time of reactor, sec

t = time, sec. t_c = time for a single circulation of coolant through a closed reactor *circuit*, t_r = time for a single passage through the *reactor* core

Q = volume rate of flow of coolant from reactor, cm³/sec

$\alpha = \dfrac{\text{weight of fluid in reactor core}}{\text{weight of fluid in the system}}$. For a constant-density fluid, this is the same as the volume ratio

λ = disintegration constant, sec⁻¹. λ_1 refers to original nuclide, λ_2 to daughter or product nuclide

ϕ = neutron flux, neutrons/(cm²)(sec)

σ = microscopic absorption cross section, cm². σ_1 and σ_{act} are the absorption and activation cross sections, respectively, of the original nuclide. σ_2 is the absorption cross section of the product nuclide.

8.1 Units

8.11 Half-life $T_{1/2} = (\ln 2)/\lambda = 0.693/\lambda$ $\lambda = 0.693/T_{1/2}$ (see Art 1.22 of Sec. 7-1).

8.12 Mean lifetime $= 1/\lambda$, sec.

8.13 1 curie $= 3.7 \times 10^{10}$ disintegrations per second—more precisely, the quantity of a radioactive material that undergoes this rate of disintegration.

8.2 Burnup, Activation, and Decay

8.21 Activation and Decay Formulas. Table 32 summarizes formulas for simple cases of activation and decay. These and other cases are given in Art. 2 of Sec. 7-1.

Cross sections and half-lives are given in Table 1 of Sec. 7-1 and Table 19 of Sec. 2.

8.22 Activity of Radioactive Parent

$$\begin{aligned} \text{Activity} &= f_1 N_0 \lambda_1 \quad \text{disint./sec} \\ &= f_1 N_0 \lambda_1/(3.7 \times 10^{10}) \quad \text{curies} \end{aligned}$$

$$\begin{aligned} \text{Activity per gram of parent nuclide initially present} &= 0.602 \times 10^{24} f_1 \lambda_1/M_1 \text{ disint./sec} \\ &= 1.63 \times 10^{13} f_1 \lambda_1/M_1 \quad \text{curies} \end{aligned}$$

8.23 Activity of Radioactive Daughter or Product

$$\begin{aligned} \text{Activity} &= f_2 N_0 \lambda_2 \quad \text{disint./sec} \\ &= f_2 N_0 \lambda_2/(3.7 \times 10^{10}) \quad \text{curies} \end{aligned}$$

$$\begin{aligned} \text{Activity per gram of original nuclide initially present} &= 0.602 \times 10^{24} f_2 \lambda_2/M_1 \text{ disint./sec} \\ &= 1.63 \times 10^{13} f_2 \lambda_2/M_1 \text{ curies} \end{aligned}$$

8.3 Fission-product Decay Heat

See Art. 3 of Sec. 7-1, also Art. 2.4 of Sec. 11 and Art. 1.7 of Sec. 2.

8.31 Way-Wigner Formula

$$\frac{P}{P_0} = 6.22 \times 10^{-2}[t^{-0.2} - (T_0 + t)^{-0.2}]$$

where P/P_0 = decay heat rate as a fraction of operating heat rate
T_0 = time fuel is in reactor, sec
t = time after shutdown or removal from reactor, sec

Approximately half the heat appears as β-particle energy and half as γ-ray energy.

8.32 Untermyer-Weills Formula. Table 33 gives the decay heat rate according to the Untermyer-Weills formula (see Art. 3.2 of Sec. 7-3).

8.33 Decay Heat of Specific Fission Nuclides. See Art. 2.4 of Sec. 11.

8.4 Coolant Activity

8.41 Activation and Activity Data. Table 34 summarizes data for dominant activities of short half-life. For data on other short-half-life nuclides, see Art. 4 of Sec. 7-1.

Table 35 gives formulas for calculating coolant activity from the data of Table 34.

8.42 Activity of Coolant. Table 35 summarizes formulas for N_2, the number of activated atoms per cubic centimeter

$$\begin{aligned} \text{Activity per cubic centimeter} &= N_2 \lambda_2 \quad \text{disint./sec} \\ &= N_2 \lambda_2/(3.7 \times 10^{10}) \quad \text{curies} \end{aligned}$$

Table 32. Simple Activation and Decay Formulas*

Case	General formula		Simplified formula for small decay or burnup (all values of λt and $\sigma\phi t < 0.1$)	
	Original atoms remaining, $f_1 = N_1/N_0$	Product or first daughter atoms, $f_2 = N_2/N_0$	Original atoms remaining, $f_1 = N_1/N_0$	Product atoms, $f_2 = N_2/N_0$
Decay of radioactive nuclide:				
Daughter stable	$\exp(-\lambda_1 t)$	$[1 - \exp(-\lambda_1 t)]$	$(1 - \lambda_1 t)$	$\lambda_1 t(1 - \frac{1}{2}\lambda_1 t) \approx \lambda_1 t$
Daughter unstable	$\exp(-\lambda_1 t)$	$\frac{\lambda_1}{\lambda_2 - \lambda_1}[\exp(-\lambda_1 t) - \exp(-\lambda_2 t)]$	$(1 - \lambda_1 t)$	$\lambda_1 t[1 - \frac{1}{2}(\lambda_1 + \lambda_2)t] \approx \lambda_1 t$
Burnup of stable or long-lived nuclide by neutron absorption:				
Product stable	$\exp(-\sigma_1\phi t)$	$\frac{\sigma_1}{\sigma_2 - \sigma_1}[\exp(-\sigma_1\phi t) - \exp(-\sigma_2\phi t)]$	$(1 - \sigma_1\phi t)$	$\sigma_1\phi t[1 - \frac{1}{2}(\sigma_1 + \sigma_2)\phi t] \approx \sigma_1\phi t$
Product unstable	$\exp(-\sigma_1\phi t)$	$\frac{\sigma_1}{\left(\sigma_2 + \frac{\lambda_2}{\phi}\right) - \sigma_1}\{\exp(-\sigma_1\phi t) - \exp[-(\sigma_2\phi + \lambda_2)t]\}$	$(1 - \sigma_1\phi t)$	$\sigma_1\phi t\{1 - \frac{1}{2}[(\sigma_1 + \sigma_2)\phi + \lambda_2]t\} \approx \sigma_1\phi t$
Activation of stable or long-lived nuclide for time t_a followed by additional decay time t_d	$\exp(-\sigma_1\phi t_a)$	$\frac{\sigma_1}{\left(\sigma_2 + \frac{\lambda_2}{\phi}\right) - \sigma_1}\{\exp(-\sigma_1\phi t_a) - \exp[-(\sigma_2\phi + \lambda_2)t_a]\}\exp(-\lambda_2 t_d)$	$(1 - \sigma_1\phi t_a)$	$\sigma_1\phi t_a[1 - \frac{1}{2}(\sigma_1 + \sigma_2)\phi t_a - \lambda_2(\frac{1}{2}t_a + t_d)] \approx \sigma_1\phi t_a$

$N_1 = f_1 N_0$ $\qquad$ $N_2 = f_2 N_0$

* Neutron-absorption formulas containing λ_2 are for the activation product, with σ_1 meaning σ_{act} in cases where absorption and activation are not synonymous.

Table 33. Decay Heat Rate* of Fission Products

Cooling time	Exposure time: 1 min	1 hr	1 day	1 week	30 days	90 days	1 year	2 years	Infinite
1 sec	0.0193 (0.0192)	0.0425 (0.0404)	0.0516 (0.0488)	0.0549 (0.0513)	0.0566 (0.0528)	0.0575 (0.0537)	0.0582 (0.0544)	0.0584 (0.0546)	0.0589 (0.0551)
10 sec	0.0132 (0.0132)	0.0355 (0.0334)	0.0446 (0.0418)	0.0479 (0.0443)	0.0496 (0.0459)	0.0505 (0.0468)	0.0512 (0.0475)	0.0514 (0.0476)	0.0519 (0.0481)
1 min	0.0050 (0.0049)	0.0234 (0.0214)	0.0325 (0.0297)	0.0358 (0.0322)	0.0375 (0.0337)	0.0384 (0.0346)	0.0390 (0.0353)	0.0393 (0.0355)	0.0397 (0.0360)
10 min	0.00052 (0.00046)	0.0089 (0.0073)	0.0174 (0.0152)	0.0207 (0.0177)	0.0224 (0.0193)	0.0234 (0.0202)	0.0240 (0.0209)	0.0242 (0.0211)	0.0247 (0.0216)
1 hr	0.00006 (0.00005)	0.0025 (0.0021)	0.0092 (0.0085)	0.0124 (0.0109)	0.0141 (0.0124)	0.0151 (0.0133)	0.0157 (0.0140)	0.0159 (0.0142)	0.0164 (0.0147)
6 hr	0.00001 (0.00001)	0.00041 (0.00040)	0.0037 (0.0034)	0.0066 (0.0056)	0.0083 (0.0071)	0.0092 (0.0080)	0.0099 (0.0087)	0.0101 (0.0089)	0.0106 (0.0094)
1 day	.....	0.00008 (0.00007)	0.0013 (0.0011)	0.0035 (0.0026)	0.0050 (0.0041)	0.0059 (0.0050)	0.0066 (0.0057)	0.0068 (0.0058)	0.0073 (0.0063)
1 week	.....	0.00001 (0.00001)	0.00018 (0.00014)	0.00089 (0.00074)	0.0019 (0.0017)	0.0027 (0.0025)	0.0033 (0.0032)	0.0035 (0.0033)	0.0040 (0.0038)
30 days	.....	.....	0.00003 (0.00003)	0.00020 (0.00020)	0.00061 (0.00061)	0.0011 (0.0011)	0.0016 (0.0016)	0.0018 (0.0018)	0.0023 (0.0023)
90 days	.....	.....	0.00001 (0.00001)	0.00005 (0.00005)	0.00018 (0.00018)	0.00041 (0.00041)	0.00076 (0.00076)	0.00090 (0.00090)	0.0014 (0.0014)
1 year	.....	.....	.....	0.00001 (0.00001)	0.00003 (0.00003)	0.00007 (0.00007)	0.00018 (0.00018)	0.00026 (0.00026)	0.00067 (0.00067)
5 years	.....	.....	.....	.....	.....	.....	.....	0.00003 (0.00003)	0.00035 (0.00035)

* The tabulated quantity is P/P_0, the decay heat rate or power as a fraction of the operating power, according to the Untermyer-Weills formula (see Art. 3.2 of Sec. 7-1). The first quantities give the total heat ratio for irradiated natural uranium. The quantities in parentheses are for the fission products alone, after allowing for heat of decay of U^{239} and Np^{239}; for practical purposes this is the heat ratio for irradiated U^{235}.

Table 34. Reactor Coolant-activity Data

Reaction	Isotopic abundance, per cent	Activation cross section σ_{act}	Reference material	Factor for determining $N_1\sigma_{act}$*	Applicable flux	Decay of product			Reactor coolants in which activity is important
						Half-life $\tau_{1/2}$	Disint. const. λ_2, sec^{-1}	Particle and energy, Mev	
$D(n,\gamma)T$	100	0.57 mb	D_2O	3.4×10^{-5}	Thermal	12.4 years	1.77×10^{-9}	β^-0.019	Heavy water
$O^{16}(n_f,p)N^{16}$	99.76	0.04 mb	H_2O D_2O	1.33×10^{-6} 1.21×10^{-6}	Virgin†	7.4 sec	0.0936	β^-3.8, 4.3, 10.3 γ6.13, 7.10	Water, heavy water, other coolants containing oxygen
$Na^{23}(n,\gamma)Na^{24}$	100	0.56 barn	Na	1.47×10^{-2}	Thermal	15.0 hr	1.28×10^{-5}	β^-1.39, 4.2 γ2.755, 1.38, 3.7	Sodium and its alloys
$K^{41}(n,\gamma)K^{42}$	6.91	1.0 barn	K	1.07×10^{-3}	Thermal	12.4 hr	1.55×10^{-5}	β^-3.6, 1.9 γ1.51	Potassium and its alloys
$A^{40}(n,\gamma)A^{41}$	99.60	0.53 barn	Air	1.33×10^{-7}	Thermal	109 min	1.06×10^{-4}	β^-1.2, 2.5 γ1.37	Air
$Bi^{209}(n,\gamma)Bi^{210}$ $\rightarrow Po^{210}$	100	19 mb	Bi	5.48×10^{-5}	Thermal	5 days 138.3 days	1.60×10^{-6} 5.80×10^{-8}	β^-1.17 α5.30, 4.5 γ0.80	Bismuth and its alloys

* For *liquid* coolants, $N_1\sigma_{act}$ is the tabulated quantity multiplied by the number of grams of reference material per cubic centimeter of coolant; if the coolant is the pure reference material, the multiplier is the density of the coolant in grams per cubic centimeter.

For *air* (0.94 per cent argon), the tabulated quantity is $N_1\sigma_1$ per cubic centimeter of air at 0°C and 76 cm Hg. This number must be adjusted for temperature, pressure, and dilution with other gases.

† Virgin flux:

$$\phi_v = \frac{3.1 \times 10^{10}\nu P}{N_H\sigma_H}$$

where ν = number of neutrons per fission
P = power density, watts/cm^3
N_H = number of hydrogen atoms per cubic centimeter
σ_H = scattering cross section of moderator for neutrons above 9 Mev.

For hydrogen, $\sigma_H \approx 1 \times 10^{-24}$ cm^2, i.e., 1 barn.

In water-moderated reactors with U^{235} (or natural uranium) as fuel (ν = 2.47)

$$\phi_v = \frac{7.6 \times 10^{10}P}{N_H}$$

At ordinary temperatures ($N_H = 6.7 \times 10^{22}$), $\phi_v = 1.14 \times 10^{12}P$.

In heavy-water moderated reactors, use the same constants but increase the calculated activity by 30 per cent.

Table 35. Activity of Reactor Coolants*

Case	Activated atoms per cubic centimeter, N_2
Closed (recirculating) systems:	
General case	$\frac{N_1\sigma_{act}\phi}{\lambda_2+\sigma_2\phi}\frac{\{1-\exp[-(\lambda_2+\sigma_2\phi)\alpha t_c]\}\{1-\exp[-(\lambda_2+\sigma_2\phi\alpha)T]\}}{\{1-\exp[-(\lambda_2+\sigma_2\phi\alpha)t_c]\}}$
Very short-lived activities† (N^{16}, etc.):	
At exit from core	$\frac{N_1\sigma_{act}\phi}{\lambda_2}[1-\exp(-\lambda_2\alpha t_c)]/[1-\exp(-\lambda_2 t_c)]$
At entrance to core	$\frac{N_1\sigma_{act}\phi}{\lambda_2}[1-\exp(-\lambda_2\alpha t_c)]\{\exp[-\lambda_2(1-\alpha)t_c]\}/[1-\exp(-\lambda_2 t_c)]$
Moderately short-lived activities‡ (A^{41}, Na^{24}, etc.)	$\frac{N_1\sigma_{act}\phi\alpha}{\lambda_2}$
Long-lived activities (most impurities):¶	
In general	$\frac{N_1\sigma_{act}\phi\alpha}{\lambda_2+\sigma_2\phi\alpha}\{1-\exp[-(\lambda_2+\sigma_2\phi\alpha)T]\}$
Special case 1. $\lambda_2 \gg \sigma_2\phi$	$N_1\sigma_{act}\phi\alpha[1-\exp(-\lambda_2 T)]$
Special case 2. $\sigma_2\phi \gg \lambda_2$	$\frac{N_1\sigma_{act}}{\sigma_2}[1-\exp(-\sigma_2\phi\alpha T)]$
Upper limit of activity§	$N_1\sigma_{act}\phi\alpha T$

	Activated atoms N_2 per cubic centimeter of effluent	Total number of accumulated undecayed effluent atoms after time T
Open (once-through) Systems:		
General case	$\frac{N_1\sigma_{act}\phi}{\lambda_2+\sigma_2\phi}\{1-\exp[-(\lambda_2+\sigma_2\phi)t_r]\}$	$\frac{N_2Q}{\lambda_2}[1-\exp(-\lambda_2 T)]$
Very short-lived activities† (N^{16}, etc.)	$\frac{N_1\sigma_{act}\phi}{\lambda_2}[1-\exp(-\lambda_2 t_r)]$	N_2Q/λ_2
Moderately short-lived activities‡ (A^{41}, Na^{24}, etc.)	$N_1\sigma_{act}\phi t_r$	N_2Q/λ_2
Long-lived activities¶ (most impurities)	$N_1\sigma_{act}\phi t_r$	$\frac{N_2Q}{\lambda_2}[1-\exp(-\lambda_2 T)]$
Upper limit§	$N_1\sigma_{act}\phi t_r$	N_2QT

* Burnup and decay of the initial nuclide are ignored in all cases. N_1 and N_2 are numbers per cubic centimeter of coolant. $N_1\sigma_1$ and λ_2 are derived from Table 34 or sources given in Art. 8.21.

† Burnup of activated nuclide during a few half-lives is assumed negligible; i.e., $\sigma_2\phi < 0.1\lambda_2$. Approach to saturation is assumed, i.e., steady operation for more than five half-lives.

‡ Burnup of activated nuclide during a few half-lives is assumed negligible; i.e., $\sigma_2\phi < 0.1\lambda_2$. Approach to saturation is assumed, i.e., steady operation for more than five half-lives. Circulation time t_c of fluid through the system (or passage time t_r through the reactor for an open system) is assumed to be short compared with the half-life, i.e., t_c or $t_r < 0.2T_{1/2}$.

¶ Burnup and decay of the activated nuclide during a few circulations are assumed negligible, i.e., $(\lambda_2+\sigma_2\phi)t_c < 0.1$. For open circuit, $(\lambda_2+\sigma_2\phi)t_r < 0.1$.

§ This upper limit of activation for a finite time is approached if burnup and decay of the activated nuclide are negligible in the time T, i.e., if $(\lambda_2+\sigma_2\phi\alpha)T < 0.1$, or for open circuit, if $\lambda_2 T < 0.1$.

9 HEALTH PHYSICS

See Sec. 7-2, by K. Z. Morgan.

9.1 Units

9.11 Units Used in Dosimetry. Table 36 describes units of radiation dose. For details, see Art. 1.2 of Sec. 7-2.

Table 36. Units of Radiation Dose

Unit	Type of radiation measured	Medium to which applicable	Basis of definition	Remarks
Roentgen, r	X-ray γ-ray of less than 3 Mev.	Air	1 esu/cm³ of air at 0°C 760 mm Hg (1 cm³ of air = 0.001293 g)	Exposure dose. Equivalent to 83.9 ergs/g air based on 32.5 ev per ion-pair, or 87.7 ergs/g based on 34 ev per ion-pair
Roentgen equivalent physical, rep	Any	Usually soft tissue	Usually 93 ergs/g (98 ergs/g based on 34 ev per ion pair)	Absorbed dose (obsolescent)
Rad	Any	Any	100 ergs/g	Absorbed dose. Standard unit for all classes of radiation work
Roentgen equivalent man, rem	Any	Any body tissue (including bone)	(Number of rads) × RBE	Standard unit for biological effects

Volume-integrated Dose. Gram-roentgen, gram-rep, and gram-rad are the doses (in r, rep, and rad, respectively) integrated to give total energy absorption over the entire body (see Art. 1.25 of Sec. 7-2).

9.12 Relative Biological Effectiveness, RBE. See Art. 1.4 of Sec. 7-2.

$$\text{RBE} = \frac{\text{Co}^{60}\gamma \text{ dose in rads to produce a biological change}}{\text{actual dose in rads to produce the same change}}$$

For RBE of heavy ionizing particles, see Table 1 of Sec. 7-2; for other particles, Table 2 of Sec. 7-2 (see also Table 37 of Sec. 1-1).

9.2 Radiation Dose from External Sources

See Art. 1.6 of Sec. 7-2.

9.21 Factors Affecting Maximum Permissible Dose. Figure 1 of Sec. 7-2 shows *linear* and *threshold* radiation-damage rates. Figures 2 and 3 show maximum permissible dose at various parts of the body.

9.22 Maximum Permissible Dose Rate. See Art. 1.6 of Sec. 7-2. Basic rate for adults 0.3 rem/week. If averaged over several years, 0.1 rem/week.

For minors under 18 and others living in the neighborhood of the controlled area, one-tenth of the above rates.

These dose rates may be increased considerably as prescribed by Figs. 2 and 3 of Sec. 7-2.

9.23 Planned emergency dose for adult male (or woman over 45), 6.25 rem. This dose may be increased considerably as prescribed by Fig. 3 of Sec. 7-2.

9.24 Calculation of Dose Rate. Table 37 of this section, based principally on Table 2 of Sec. 7-2, summarizes the particle fluxes corresponding to maximum permissible dose rate. Footnotes indicate other sources of data.

Table 37. Flux for 7.5 mrem/hr at Surface of Body (0.3 rem per 40-hr Week)

Type of radiation	RBE	Approximate flux, particles/(cm²)(sec)
X and γ rays	1	4,200/E (Mev)
Thermal neutrons	(2.5)	1,930 (2,000*)
Fast neutrons*	(10)	Mev: >3, 3, 2, 1, 0.5, 0.1, 0.01 Flux: 30, 30, 40, 60, 80, 200, 1,000
Beta rays and electrons†	1	4,340/S_a, S_a from Table 3 of Sec. 7-2
Alpha particles†	10	400/S_a, S_a from Table 3 of Sec. 7-2
Protons	10	1.3×10^7/(stopping power from Fig. 4 of Sec. 7-2)
Heavy ions	20	Oxygen ion: 6.5×10^6/(stopping power from Fig. 4 of Sec. 7-2)

* International values (see Table 4 of Sec. 7-2). Actual biological dose and energy deposition in tissue as a function of depth are given by Snyder, Figs. 5 to 13 of Sec. 7-2. Table 4 of Sec. 7-2 gives maximum permissible neutron flux based on these curves.

† Flux is also given for electrons by 1.3×10^8/(stopping power) and for α particles by 1.3×10^7/(stopping power). Obtain stopping power from Fig. 4 of Sec. 7-2. The results are only approximately equal.

Equations (12) to (19) with Table 3 permit calculation of dose in soft tissue or bone at any depth, or averaged over the range of a particle of given initial energy.

9.25 Natural Background and Common X-ray Exposures. See Tables 5 and 6 of Sec. 7-2.

9.3 Maximum Permissible Internal Dose

The maximum permissible concentration in air or water is determined by the maximum permissible body burden for the particular nuclide to produce 0.3 rem/week at the critical organ. Article 1.7 of Sec. 7-2 describes methods of calculation for specific nuclides, for both continuous and single exposures. For periods not exceeding a few months, it is usually safe to use the following "general maximum permissible concentrations" (from Table 7 of Sec. 7-2).

β or γ emitters	10^{-9} μc/ml air	10^{-7} μc/ml water
α emitters	5×10^{-12} μc/ml air	10^{-7} μc/ml water

Dose from Internally Absorbed Discrete Particles. See Eqs. (33) to (36) of Sec. 7-2.

10 NUCLEAR RADIATION SHIELDING

See Sec. 7-3, by E. P. Blizard.

Nomenclature

B_a = energy absorption build-up factor, a function of geometry, material, energy, and number of relaxation lengths (Table 7 of Sec. 7-3)
B_r = dose build-up factor, a function of geometry, material, energy, and number of relaxation lengths (Tables 4 and 5 of Sec. 7-3)
D = dose rate (including build-up), rad/hr
E_0 = photon energy of source, Mev.
H = rate of heat deposition, ergs/(sec)(g)
H' = rate of heat deposition, watts/g
R = distance from point source, cm
S = source strength for point source, photons/sec
t = thickness of attenuating material for point source, cm
x = thickness of attenuating material for collimated plane source, cm
μ = linear attenuation coefficient of uncollided flux, cm^{-1}
ρ = density of material, g/cm^3

μ/ρ = total mass attenuation coefficient, a function of material and energy (Table 1 of Sec. 7-3), cm^2/g

μ_a/ρ = energy-absorption mass attenuation coefficient, a function of material and energy (Table 2 of Sec. 7-3), cm^2/g

Γ = uncollided gamma flux at penetration t or x, photons/(cm^2)(sec)

Γ_0 = incident gamma flux for plane source, photons/(cm^2)(sec)

10.1 Relaxation Length, Half-thickness, and Tenth-thickness

The thickness of material to attenuate a γ dose rate by a factor of 2, e, or 10 is convenient for rule-of-thumb estimate of attenuation. The thicknesses are called the relaxation length, half-thickness, and tenth-thickness, respectively, and are interconvertible in accordance with Table 38. These characteristic thicknesses are a function of γ energy and also vary with actual thickness because of the build-up factor.

Table 38. Relaxation Length, Half-thickness, and Tenth-thickness

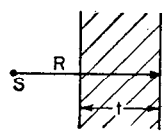
Relaxation lengths	Half-thicknesses	Tenth-thicknesses
1	1.443	0.4343
0.6931	1	0.3010
2.303	3.322	1

10.2 γ-ray Attenuation from Point and Plane Sources

Step 1. Calculate uncollided flux Γ

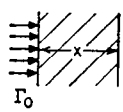
Point source:

$$\Gamma = \frac{S}{4\pi R^2} \times e^{-\mu t} \qquad \text{Eq. (2) of Sec. 7-3}$$



Plane source:

$$\Gamma = \Gamma_0 e^{-\mu x} \qquad \text{Eq. (2}a\text{) of Sec. 7-3}$$



Determination of μ. Table 1 of Sec. 7-3 gives (μ/ρ). $\mu = (\mu/\rho)\rho$. Table 21 of Sec. 7-3 gives both (μ/ρ) and μ for selected heavy elements in a more limited range than Table 1.

*Number of relaxation lengths** = μt or μx.

Step 2. Calculate γ-radiation dose rate D or rate of heat deposition H at penetration t for a point source

$$D = 5.767 \times 10^{-5}(\mu_a/\rho)E_0\Gamma B_r \qquad \text{rad/hr} \qquad \text{Eqs. (3) and (4) of Sec. 7-3}$$

Simplified solution for tissue:

$$D = \frac{\Gamma B_r}{\text{(Table 3, column 2)}} = \frac{E_0\Gamma B_r}{\text{(Table 3, column 3)}} \qquad \text{rad/hr}$$

$$H = 1.602 \times 10^{-6}(\mu_a/\rho)E_0\Gamma B_a \qquad \text{ergs/(sec)(g)} \qquad \text{Eq. (5) of Sec. 7-3}$$

$$H' = 1.602 \times 10^{-13}(\mu_a/\rho)E_0\Gamma B_a \qquad \text{watts/g}$$

Determination of E_0. See Art. 2 of Sec. 7-3, also Tables 1 and 2 of Sec. 7-1.

* Relaxation length (thickness to produce attenuation by factor e) = $1/\mu$.

Determination of (μ_a/ρ), B_r, and B_a. (μ_a/ρ) from Table 2 of Sec. 7-3; B_r from Table 4 of Sec. 7-3 (for point source) and Table 5 (for plane source); B_a from Table 7 (for point source).

10.3 γ-Ray Attenuation, Special Cases

10.31 Lead Shield for Co^{60} and Cs^{137} Sources. Thickness for 7.5 mrem/hr at outer surface (see Table 6 of Sec. 7-3).

10.32 Approximate Unshielded γ Dose from a Point Source. See Art. 9.2 of Sec. 11.

$$D \approx 7CE_0/(R')^2$$

where D = dose rate, r/hr $\approx$ rad/hr
C = activity of point source, curies
E_0 = photon energy, Mev
R' = distance from source, ft

10.33 Composite Shields. Shields containing a homogeneous mixture of elements and laminated shields (see Art. 1.4 of Sec. 7-3).

10.4 Sources of γ and X Rays

The energy spectrum of prompt γ rays from fission of U^{235} is given in Table 8 of Sec. 7-3; the decay energy of fission products as a function of time and energy groups in Fig. 4 of Sec. 7-3; and γ energy from neutron capture in Table 9 of Sec. 7-3.

10.5 Neutron Attenuation Principles

10.51 Biological Dose from Point Source of Fission Neutrons. *Hydrogenous Shields.* Equations (13) and (14) in Art. 3.4 of Sec. 7-3 give solutions in terms of shield thickness, hydrogen density and thickness (Fig. 5), and removal cross sections (Table 12 or Fig. 6). Separate solutions are given for nonaqueous and aqueous shields.

Nonhydrogenous shields (less than 50 atomic per cent hydrogen). The dose is calculated by Eq. (15) or (16) as a function of biological effect of neutrons (Table 13), distance from source, shield thickness, and total removal cross section.

10.6 Sources of Neutrons

The U^{235} fission spectrum of prompt neutrons is given in Table 15 of Sec. 7-3, delayed neutron data in Table 16 (see also Table 39 of Sec. 1-1), and α- and photoneutron sources in Tables 17 and 18.

10.7 Geometry

Article 5 of Sec. 7-3 gives methods of calculating γ and neutron attenuation for various geometries, with curves and tables of pertinent functions.

10.8 Shield Materials

Article 6 of Sec. 7-3 contains nuclear and engineering data of common shielding materials. For list of tables, see Table 9 of Sec. 1-1.

11 PHYSICS OF REACTOR CONTROL

See Sec. 8-1, by S. Krasik. The section is based on one-group diffusion theory for a bare reactor.*

* Two-group theory for a bare reactor and the effect of a reflector on neutron lifetime are discussed in Arts. 13 and 14 of Sec. 6-2.

Nomenclature

B^2 = buckling (geometric), cm^{-2}
k = multiplication constant in infinite medium, dimensionless; k_{eff} is "k effective"; k_{ex} is "excess k"
L = diffusion length, cm
T = solutions of the reactor kinetics equation, sec; T_s is the stable period (e-folding time) after transients have disappeared
M = multiplication
S = source strength, neutrons/sec
V = volume of reactor core, cm^3
v = neutron velocity, cm/sec
β = fraction of delayed neutrons per neutron emitted in fission; β_i refers to ith group
Λ = effective neutron lifetime, sec. l^* is frequently used for Λ
λ = decay constant of delayed neutrons, sec^{-1}; λ_i refers to ith group.
ρ = reactivity, dimensionless
Σ_a = macroscopic absorption coefficient, cm^{-1}
τ = mean life of delayed neutrons, sec; τ_i refers to ith group; $\bar{\tau}$ is average mean life
ϕ = neutron flux, neutrons/(cm^2)(sec); ϕ_0 is initial flux

Table 39. Delayed-neutron Data for U^{235}*

Group	$\tau_{1/2}{}^i$ half-life, sec	τ_i	λ_i	$100\beta_i/\beta$ percentage of delayed neutrons	β_i	$\beta_i\tau_i$
1	55.72	80.39	0.0124	3.3	0.00021	0.0170
2	22.72	32.78	0.0305	21.9	0.00140	0.0459
3	6.22	8.97	0.111	19.6	0.00125	0.0113
4	2.30	3.32	0.301	39.5	0.00253	0.0084
5	0.610	0.88	1.14	11.5	0.00074	0.00065
6	0.230	0.33	3.01	4.2	0.00027	0.000089
Totals.....	.....	.....	.....	100.0	0.0064	0.0833

* Data of Keepin, Wimett, and Zeigler, Tables 2 to 4 of Sec. 8-1.

11.1 Delayed-neutron Data

Table 1 of Sec. 8-1 gives the data of Hughes et al. for thermal fission of U^{235} and Tables 2 to 4 give later data by Keepin, Wimett, and Zeigler for thermal and fast fission of Th^{232}, U^{233}, U^{235}, U^{238}, Pu^{239}, and Pu^{240}. Table 39 of this article summarizes data for U^{235} from Tables 2 to 4 of Sec. 8-1; and numerical values in this article are based on these data.

Table 40. Reactivity Conversion Factors for U^{235}*

Data of Keepin and Wimett				Data of Hughes, Dobbs, Cahn, and Hall			
Reactivity, ρ	Inhours	Dollars	Cents	Reactivity, ρ	Inhours	Dollars	Cents
1	$\sim 4.35 \times 10^4$	156	1.56×10^4	1	$\sim 3.84 \times 10^4$	132	1.32×10^4
$\sim 2.3 \times 10^{-5}$	1	$\sim 3.6 \times 10^{-3}$	~ 0.36	$\sim 2.6 \times 10^{-5}$	1	$\sim 3.45 \times 10^{-3}$	~ 0.345
0.0064	~ 280	1	100	0.00755	290	1	100
6.4×10^{-5}	~ 2.8	0.01	1	7.55×10^{-5}	2.9	0.01	1

* One dollar of reactivity corresponds to prompt critical, i.e., ρ equal to β.

Reactivity in inhours = ρ/(ρ for stable period of 1 hr)

In both cases ρ may be calculated by the inhour equation (Art. 11.41 below), but the table is based on the approximation that ρ for a stable period of 1 hr is equal to $\beta\bar{\tau}/3{,}600$. For long periods, reactivity in inhours is approximately $3{,}600/T_s$.

11.2 Reactivity

$$k_{eff} = \frac{\text{neutrons born in } (n+1)\text{th generation}}{\text{neutrons born in } n\text{th generation}} = \frac{k}{1 + L^2B^2}$$ Art. 1.2 of Sec. 8-1

$$\rho = \frac{k_{eff} - 1}{k_{eff}} = \frac{k_{ex}}{k_{eff}}$$

$$k_{eff} = 1 + k_{ex} = \frac{1}{1 - \rho} \qquad k_{ex} = k_{eff} - 1 = \frac{\rho}{1 - \rho}$$

11.3 Effective Neutron Lifetime*

$$\Lambda = 1/[v\Sigma_a(1 + L^2B^2)] \text{ sec}$$

where v = neutron velocity, cm/sec
See Art. 1.2 of Sec. 8-1, also Eq. (245) of Sec. 6-2.

11.4 Reactor Kinetics Equation

It is assumed that there is no loss of delayed neutrons.

11.41 Inhour Equation

$$\rho = \frac{k_{eff} - 1}{k_{eff}} = \frac{\Lambda}{Tk_{eff}} + \sum_i \frac{\beta_i}{1 + \lambda_i T}$$ Eq. (4) of Sec. 8-1; also Eq. (241) of Sec. 6-2

$$= \frac{1}{1 + \frac{\Lambda}{T}}\left[\frac{\Lambda}{T} + \sum_i \frac{\beta_i}{1 + \lambda_i T}\right]$$ Eq. (5) of Sec. 8-1

For U^{235}, the data of Keepin and Wimett give:

$$\rho = \frac{1}{1 + \frac{\Lambda}{T}}\left[\frac{\Lambda}{T} + \frac{0.00021}{1 + 0.0124T} + \frac{0.00140}{1 + 0.0305T} + \frac{0.00125}{1 + 0.111T} + \frac{0.00253}{1 + 0.301T} + \frac{0.00074}{1 + 1.14T} + \frac{0.00027}{1 + 3.01T}\right]$$

11.42 Guide to Solution of the Inhour Equation. See Art 1.2 of Sec. 8-2.

The Stable Reactor Period is the e-folding time after transients have disappeared.

For positive reactivity, T_s can have any positive value.

For negative reactivity, T_s can have any negative value numerically greater than τ_1 (80.39 sec for U^{235}).

The inhour equation is solved by trial, making use of simplified approximate formulas (below) where applicable.

The Six Transient Roots. These are all real and negative. They are usually solved by trial, and are located by the sequence

$$-80.39 < T_1 < -32.78 < T_2 < -8.97 < T_3 < -3.32 < T_4 < -0.88 < T_5 < -0.33 < T_6 < -\Lambda$$

11.43 Approximate Formulas for Stable Period with Positive Reactivity. See Art. 1.3 of Sec. 8-1.

Very long periods (small reactivity): $\rho < 0.0005$, $T_s > 200$ sec (useful also as first approximation for shorter periods—the approximate value of T_s is too large).

* The following is for bare-reactor one-group theory. The contribution of slowing down to neutron lifetime (two-group theory) is discussed by Dietrich in Arts. 13.3 and 14.2 of Sec. 6.2.

$$T_s = \frac{1}{\rho}(\Lambda + \beta\bar{\tau}) \approx \frac{\beta\bar{\tau}}{\rho} \qquad \text{(see Art. 1.32 of Sec. 8-1)}$$

For U^{235},

$$T_s = \frac{1}{\rho}(\Lambda + 0.0833) \approx \frac{0.0833}{\rho}$$

Very short periods (reactivity exceeding prompt critical by an amount numerically equal to at least several times Λ).

$$T_s = \frac{\Lambda}{k_{eff}(\rho - \beta)} = \frac{\Lambda(1 - \rho)}{(\rho - \beta)}$$

See Art. 1.33 of Sec. 8-1; also Eq. (242) of Art. 6-2.

11.44 Approximate Formulas for Stable Period with Negative Reactivity. See Art. 1.5 of Sec. 8-2.

Very long periods (small negative reactivity), T_s negative but numerically greater than 200 sec. Same as for positive reactivity.

Shorter Periods. The period is always numerically larger than the mean life of the longest-lived delayed neutrons. For U^{235} this is 80.39 sec. The limit $T_s = -80.39$ sec is approached for very large negative reactivity.

11.45 Rough Approximation of Time Behavior for Step Change of Reactivity

1. Prompt jump (or drop)

$$\frac{\phi}{\phi_0} \approx \frac{\beta(1 - \rho)}{\beta - \rho}$$

occurring in a small fraction of a second. See Art. 1.6 of Sec. 8-2 and Eq. (244) of Sec. 6-2.

2. The flux increases from the level of the prompt jump, with a stable period T_s calculated from Eq. (4) of Sec. 8-1 or from approximations.

11.5 Steady-state Subcritical Reactor ($k_{eff} < 1$) with Source

See Art. 1.2 of Sec. 8-2, Art. 1.1 of Sec. 8-3, and Art. 13.1 of Sec. 6-2

Source multiplication: $M = \dfrac{1}{1 - k_{eff}}$

Total neutron production: MS n/sec

Total neutron population:

$$MS\Lambda = \frac{S\Lambda}{1 - k_{eff}} \quad \text{neutrons}$$

Neutron flux in core: $\dfrac{MS\Lambda v}{V} = \dfrac{S\Lambda v}{V(1 - k_{eff})}$ n/(cm²)(sec)

12 FLUID PROPERTIES

See Sec. 9-1, by Wayne H. Jens.

12.1 Units

Nonsystematic units (see Art. 1.15 of Sec. 1-3) are used in fluid-flow and heat-transfer calculations.

12.11 Fluid Flow. The units are foot, second, pound-mass, and pound-force.

12.12 Heat Flow. The units are foot, hour, pound-mass, Btu, and degree Fahrenheit.

12.13 Fluid Properties. Properties are expressed in Sec. 9-1 in the same units as used for heat flow, i.e., foot, hour, pound-mass, Btu, and degree Fahrenheit.

12.2 Dimensionless Numbers or Moduli

Dimensionless numbers must be calculated from properties given in consistent units. Thus in fluid-flow problems velocity, which is expressed in feet per second for all other calculations, must be expressed in feet per hour in calculating dimensionless numbers such as Reynolds number from data of Sec. 9-1.

Viscosity data from other sources are often expressed in other units. If it is convenient to express the other quantities in consistent units (as in the metric system), the data can be used directly; otherwise it is suggested that Table 17 of Sec. 1-2 be used to effect conversion.

The most frequently used dimensionless numbers are:

$$\text{Reynolds number, Re} = \frac{DV\rho}{\mu} = \frac{DG}{\mu}$$

$$\text{Nusselt number, Nu} = \frac{hD}{k}$$

$$\text{Prandtl number, Pr} = \frac{c_p\mu}{k}$$

For meaning of symbols, see nomenclature tables in Arts. 13 and 14. Table 3 of Sec. 9-2 gives a list of dimensionless numbers.

Density of Water

Table 41 gives the density of water in grams per cubic centimeter at atmospheric pressure from 0 to 100°C. The density maximum of 0.99997 g/cm³ at approximately 4°C corresponds to 1 g/ml exactly (see definition of milliliter in Table 6 of Sec. 1-3).

For temperatures above 100°C, Table 19 of Sec. 9-1 gives the specific volume v in cubic feet per pound over a range of temperature (°F) and pressure (psi). The density is given by:

$$\rho = \frac{1}{62.428v} = \frac{0.01602}{v} \qquad \text{g/cm}^3$$

The density of heavy water is given in Table 27 of Sec. 9-1.

Table 41. Density of Water, g/cm³

°C	0	1	2	3	4	5	6	7	8	9
0	0.99 984	990	994	996	997	996	994	990	985	978
10	970	961	950	938	925	910	895	878	860	841
20	821	799	777	754	730	705	679	652	624	595
30	565	534	503	471	438	404	369	333	297	260
40	222	183	144	104	063	021	*979	*937	*893	*849
50	0.98 804	759	712	666	618	570	522	472	422	372
60	321	269	217	164	110	056	002	*947	*891	*835
70	0.97 778	720	662	604	545	486	426	365	304	242
80	180	118	055	*991	*927	*862	*797	*731	*665	*598
90	0.96 531	464	396	327	258	189	119	049	*978	*907
100	0.95 835									

13 FLUID FLOW

See Sec. 9-2, by Charles F. Bonilla.

Nomenclature

c_p = specific heat at constant pressure, Btu/(lb$_M$)(°F)
D = pipe diameter, ft
D_e = equivalent diameter of noncircular duct, ft
e = surface roughness, ft
f_F = Fanning friction factor, dimensionless
G = mass velocity. In fluid flow, units are lb$_M$/(ft^2)(sec); in heat flow, units are lb$_M$/(ft^2)(hr)
g_c = constant numerically equal to gravitational acceleration, 32.17 ft/sec^2. Usually regarded as a dimensional constant (ft)(lb$_M$)/(sec^2)(lb$_F$). Sometimes regarded as dimensionless
V = average velocity, ft/sec
v = specific volume, ft^3/lb
w = rate of mass flow, lb/sec
β = volumetric coefficient of thermal expansion, (°F)$^{-1}$
K = number of velocity heads lost, dimensionless
L = length, ft
ΔP = pressure drop, lb$_F$/ft^2
Q = volume rate of fluid flow, ft^3/sec
S = cross-sectional area of fluid stream, ft^2
μ = viscosity = absolute viscosity = dynamic viscosity, lb$_M$/(hr)(ft). μ_w is viscosity of fluid at wall temperature, μ_b is viscosity at bulk temperature
ρ = density, lb$_M$/ft^3

13.1 General Equations

13.11 Continuity Equation

$$G = V\rho \qquad w = Q\rho = V\rho S = GS \qquad \text{Eq. (44) of Sec. 9-2}$$

13.12 Velocity Pressure

$$\text{Velocity pressure} = \frac{\rho V^2}{2g_c} = \frac{G^2}{2\rho g_c} = \frac{G^2 v}{2g_c}$$

13.2 Friction in a Straight Pipe

13.21 Pressure Drop by Friction

$$\Delta P = 4f_F \cdot \frac{L}{D_e} \cdot \frac{\rho V^2}{2g_c} = 4f_F \cdot \frac{L}{D_e} \cdot \frac{G^2}{2\rho g_c} \qquad \text{[See Eq. (28) of Sec. 9-2]}$$

f_F is the *Fanning factor*, given in Fig. 2 of Sec. 9-2. Many authors eliminate the 4 by using a *friction factor* f equal to $4f_F$.

13.22 Streamline Flow, Re < 2,100. See Art. 3 of Sec. 9-2.

For isothermal flow in long channels,

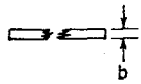
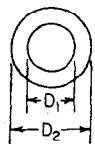
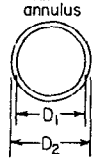
$$f_F = 16/\text{Re} \qquad \Delta P = 4f_F \cdot \frac{L}{D_e} \cdot \frac{\rho V^2}{2g_c} = \frac{32V\mu L}{D_e^2 g_c}$$

For short channels, see Table 2 of Sec. 9-2.

D_e *for streamline flow* is given in Table 42 below (see Table 1 and Art. 3.12 of Sec. 9-2, in which other cross sections are also discussed).

For nonisothermal flow (fluid and walls at different temperatures), calculate ΔP (or f_F) as above, and multiply by $(\mu_w/\mu_b)^{0.23}$ if the fluid is being cooled, and $(\mu_w/\mu_b)^{0.32}$ if the fluid is being heated. (See Art. 3.2 of Sec. 9-2 which also gives alternate treatments.)

Table 42. Streamline Flow

	Circular	Square	Parallel slot	Annulus	Narrow annulus
Shape					
D_e^2	D^2	$1.12a^2$	$\frac{8}{3}b^2$	$D_2^2 + D_1^2 - \frac{D_2^2 - D_1^2}{\ln D_2/D_1}$	$\frac{2}{3}(D_2 - D_1)^2$

13.23 Turbulent Flow, Re > 3,500. See Art. 4.2 of Sec. 9-2.

Fanning Factor f_F. Calculate Re, estimate e/D from Art. 4.2 of Sec. 9-2, read f_F from Fig. 2 of Sec. 9-2.

Examples:

e for drawn tubing, 0.000,005 ft; for commercial steel pipe, 0.00015 ft. f_F in reactor applications is typically 0.006, but may range from 0.015 to 0.003 or less.

Isothermal Flow. Apply the equation of Art. 13.21.

Nonisothermal Flow. If fluid and wall are at different temperatures, calculate f_F as above, but

1. for gases, use μ and ρ at the average of bulk gas temperature and wall temperature (for alternate method see Art. 4.21 of Sec. 9-2).
2. for liquids, use bulk liquid temperature and multiply f_F by 0.98 $(\mu_w/\mu_b)^{0.13}$.

The bulk density is used in calculating $\rho V^2/2g_c$.

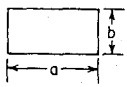
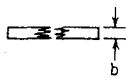
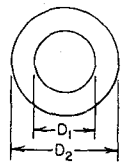
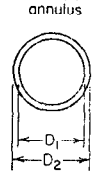
D_e for Turbulent Flow. See Art. 4.2 of Sec. 9-2. For a general approximation,

$$D_e = \frac{4 \times \text{area of cross section}}{\text{wetted perimeter}}$$

Table 43 (from Art. 4.2 of Sec. 9-2) gives D_e for special cases; the formulas conform exactly to the above formula in all cases except for the annulus.

Compressible Fluids. If the pressure drop is small compared with the total pressure, use average properties; otherwise see Art. 4.22 of Sec. 9-2.

Table 43. Special Cases

	Circular	Rectangular	Square	Parallel slot	Annulus	Narrow annulus
Shape						
D_e	D	$\frac{2ab}{a+b}$	a	$2b$	$D_2 - \frac{D_2^2 - D_1^2}{2D_2 \ln D_2/D_1}$	$D_2 - D_1$

13.3 Friction by Change of Direction or Pipe Cross Section in Turbulent Flow

$$\Delta P = K \frac{\rho V^2}{2g_c}$$

13.31 Pipe Fittings and Bends. See Table 4 of Sec. 9-2.

Table 44. K for Sudden Change of Cross Section

S_1/S_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Enlargement	1	0.81	0.64	0.49	0.36	0.25	0.16	0.09	0.04	0.01	0
Contraction*	(0.4)	0.36	0.34	0.31	0.27	0.22	0.16	0.11	0.05	0.02	0

* K for contraction varies appreciably with conditions. The tabulated values are the lowest reported (Weisbach) for a sharp-cornered contraction—to be conservative 0.1 should be added throughout. For rounded edges the values are much lower.

13.32 Sudden Change of Cross Section. Table 44 gives K for sudden enlargement and sudden contraction of a pipe.

$$S_1/S_2 = \text{(smaller cross-sectional area)/(larger cross-sectional area)}$$

K is to be used with the higher velocity, i.e., the velocity in the smaller of the two cross sections. For more accurate treatment, see Art. 4.3 of Sec. 9-2.

For a *well-rounded entrance* to a contraction, $K = 0.05$.

13.33 Orifices and Nozzles in Pipes. The over-all pressure-loss factor, based on the velocity at the orifice or nozzle, is given in Table 45.

Table 45. K for Orifices and Nozzles in Pipes

Orifice or nozzle diam. / Pipe inside diameter	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
K for sharp-edged orifice	2.68	2.62	2.38	2.16	1.87	1.50	1.10	0.66	0.40
K for nozzle	0.98	0.92	0.83	0.70	0.56	0.41	0.26	0.13	0.04

In general, $K = 1/C_D^2$, where C_D is the discharge coefficient of the orifice system, including a sufficient length of pipe to permit contraction and subsequent expansion of the stream.

13.4 External Flow

13.41 Flow across Tube Banks. For more than five rows of tubes, K per row (based on maximum velocity): 0.72 for triangular pitch, 0.32 for rectangular pitch. Article 8.2 of Sec. 9-2 gives a more accurate treatment.

13.42 Flow through Beds of Particles. See Art. 8.3 of Sec. 9-2.

13.5 Pressure Changes Other Than Friction Losses

13.51 Acceleration. The subscripts 1 and 2 refer to the upstream and downstream side, respectively. A positive value of ΔP means a pressure drop in the direction of flow, a negative value means a pressure gain. Acceleration causes a loss (ΔP positive), deceleration causes a gain (ΔP negative).

Pressure Drop Due to Change of Velocity at Constant Density

$$\Delta P = \frac{\rho(V_2^2 - V_1^2)}{2g_c} = \frac{\rho V_1^2}{2g_c}\left[\left(\frac{S_1}{S_2}\right)^2 - 1\right] = \frac{\rho V_2^2}{2g_c}\left[1 - \left(\frac{S_2}{S_1}\right)^2\right]$$

Since this drop for a closed system usually does not exceed one high-velocity head, it is usually ignored in rough calculations.

Acceleration Pressure Drop Due to Change of Density in a Channel of Constant Cross Section

$$\Delta P = \frac{\rho_2 V_2^2 - \rho_1 V_1^2}{g_c} = \frac{G}{g_c}(V_2 - V_1)$$

For a more exact formula see Sec. 9-2, Eq. (30). This drop can also generally be disregarded except for a boiling liquid.

Change of Velocity Distribution. In rough calculations, it is usual to ignore the small changes in pressure that accompany velocity redistribution across a channel.

13.52 Difference of Elevation. See Eqs. (15) and (16) of Sec. 9-2.

13.6 Total Pressure Drop

The total pressure drop between two points is the algebraic sum of losses by friction, and pressure drop by acceleration and difference of elevation. If parameters vary, sections are subdivided and formulas are evaluated over the subdivisions, using average values for the parameters. In some cases analytic solutions are available for cases of varying parameters (see Art. 4 of Sec. 9-2).

14 HEAT TRANSFER

See Sec. 9-3, by Charles F. Bonilla.

Nomenclature

A = area normal to heat flow in slab geometry, ft^2
a = width, ft
H = heat generation per unit volume per unit time, Btu/(ft^3)(hr). H is assumed to be constant unless otherwise stated
h_f = film coefficient, Btu/(ft^2)(hr)(°F)
h_R = radiation coefficient, Btu/(ft^2)(hr)(°F)
h_{GR} = gas radiation coefficient, Btu/(ft^2)(hr)(°F)
k = thermal conductivity, Btu/(ft)(hr)(°F). k is assumed to be constant
L = length of cylinder, ft
q = total heat flow per unit time, Btu/hr. It is assumed that heat flow is normal to the surface in slab geometry and radial in cylindrical and spherical geometries
r = radius, ft
T = absolute temperature, °K
t = temperature, °F
x = distance from left face of a slab or plate, ft

Subscripts: 1, 2, 3, — = first, second, third,—layer. For quantities which are constant for a layer (a, H, k) the subscript refers to the entire layer. For quantities which vary across a layer (q/A, $q/2\pi L$, $q/4\pi$, r, and t) the subscript refers to the values at the left side of a slab layer or the inner radius of a cylindrical or spherical layer (Tables 46 to 49).

Surface film coefficients and fluid temperatures take the same subscript as the adjacent wall temperature, the fluid temperature being primed to distinguish it from the wall temperature.

Variables (x, r, and t) are indicated by absence of a subscript.

14.1 Steady-state Conduction

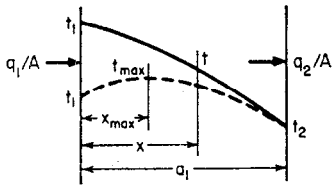
Conduction formulas for important simple cases are collected and tabulated in this article. For other formulas and for unsteady-state conduction, see Arts. 2 and 7 of Sec. 9-3.

Algebraic Signs. In all tables in this section, the positive direction of distance and heat flow is to the right in slab geometry, and radially outward in cylindrical and spherical geometries. If heat flow is in the negative direction, algebraic formulas remain unchanged, but when numerical values are substituted, the negative direction is recognized by change of sign. Negative values arising in the course of computation must be retained.

14.11 Conduction in Homogeneous Solids with Internal Heat Generation. *Slab or Plate.* Table 46 gives formulas for heat flow normal to the surface.

Table 46. Conduction in Homogeneous Slabs or Plates

Nomenclature and General Procedure*



Calculate q_1/A if not already known:
(a) If q_2 is known, $q_1/A = q_2/A - H_1a_1$
(b) If $t_1 - t_2$ is known, q_1/A is given in each particular case by the first equation for that case.
k_1 = thermal conductivity
H_1 = heat generation per unit time per unit volume
For sign conventions see text.

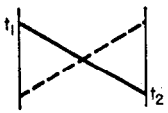
General Solution†

See above figure.
Criterion for an internal temperature maximum:
cooling from both sides,
or $a_1H_1 > (-q_1/A) > 0$

$$q_1/A = (t_1 - t_2)k_1/a_1 - H_1a_1/2$$
$$t_1 - t_2 = (q_1/A + H_1a_1/2)a_1/k_1$$
$$t_1 - t = (q_1/A + H_1x/2)x/k_1$$
$$t_{max} - t_1 = (q_1/A)^2/(2H_1k_1)$$

Heat Through-flow with No Internal Heat Generation

See Eq. (8) of Sec. 9-3.
$H_1 = 0$



$$q/A = (t_1 - t_2)k_1/a_1$$
$$t_1 - t_2 = (q_1/A)a_1/k_1$$
$$t_1 - t = (q_1/A)x/k_1$$
Highest temperature: t_1 or t_2 at $x = 0$ or $x = a_1$, respectively.

Internal Heat Generation with No External Heat Input

Case 1. Heat removed at right-hand surface

See Eq. (9) of Sec. 9-3.

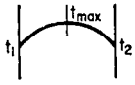
$$q_1/A = 0$$
$$t_1 - t_2 = H_1a_1^2/2k_1$$
$$t_1 - t = H_1x^2/2k_1$$
$$x_{max} = 0 \qquad t_{max} = t_1$$

Case 2. Heat removed at left-hand surface

This case becomes redundant if it is regarded as a mirror reflection of Case 1.

$$q_1/A = -H_1a_1$$
$$t_2 - t_1 = H_1a_1^2/2k_1$$
$$t - t_1 = H_1(a_1 - x/2)x/k_1$$
$$x_{max} = a_1 \qquad t_{max} = t_2$$

Case 3. Heat removed at both surfaces, surface temperatures equal



$$q_1/A = -H_1a_1/2$$
$$t - t_1 = H_1(a_1 - x)x/2k_1$$
$$x_{max} = a_1/2 \qquad t_{max} - t_1 = H_1a_1^2/8k_1$$

---

* Solutions in this table are for uniform heat generation. For exponential heat generation of the form

$$H = \alpha \exp(-\mu_\alpha x) + \beta \exp(-\mu_\beta x)$$

the general equation is

$$t_1 - t = \left\{(q_1/A)x + \frac{\alpha}{\mu_\alpha^2}[\mu_\alpha x + \exp(-\mu_\alpha x) - 1] + \frac{\beta}{\mu_\beta^2}[\mu_\beta x + \exp(-\mu_\beta x) - 1]\right\} \Big/ k_1$$

To obtain $t_1 - t_2$ write a_1 for x.

† To take into account unequal surface cooling or heating conditions of a single-layer slab, use Table 49, but reject all terms applying to second and third layers.

Hollow Cylinder. Hollow-cylinder formulas contain terms of the type $\ln r_2/r_1$. When r_2/r_1 is not large, it is convenient to write

$$\ln r_2/r_1 = F \times \frac{\text{radial thickness of tube wall}}{\text{average radius of tube}} = F\,\frac{r_2 - r_1}{(r_2 + r_1)/2} = F\,\frac{2(r_2/r_1 - 1)}{r_2/r_1 + 1}$$

r_2/r_1	1	1.5	2	3	4
F	1	1.01	1.04	1.10	1.15

Table 47 gives formulas for radial heat flow. When $r_2/r_1 < 2$, it is also satisfactory to use slab formulas, with a_1 equal to the wall thickness of the tube and the flow area equal to the cylindrical surface at average radius, i.e., $\pi(r_1 + r_2)L$.

Sphere. Table 48 gives formulas for radial heat flow in spheres.

14.12 Conduction in Multilayer Solids with Internal Heat Generation and Convection Cooling. Table 49 gives solutions for the temperature differences across layers for multilayer slabs, cylinders, and spheres, with heat generation uniform in each layer and with convection cooling on both sides of the assembly. Solutions are given for three-region solids, but they can be extended to any number of regions by including terms for additional regions.

Procedure. The specified parameters are evaluated in sequence for the geometry in question. Usually the temperature difference Δt_{tot} between the coolant streams is known* and in this case q'_1 is determined from the second of the two formulas for q'_1. If the heat flow (at either surface) is known instead of the temperature difference, q'_1 is determined from the first of the two formulas.† If the left-hand or inner surface is cooled, the numerical value of q'_1 is negative.

Cylindrical or Spherical Assembly with Solid Central Region. For a solid central region the solution is modified as follows:

1. Continue to designate the solid inner core as region 1, with radius r_2.
2. For a cylinder $f_1 = H_1 r_2^2/2$ and $m_1 = H_2 r_2^2/4k_1$. For a sphere $f_1 = H_1 r_2^3/3$ and $m_1 = H_1 r_2^2/6k_1$. The parameters g_1, l_{f0}, l_1, and q'_1 have no meaning.
3. Calculate f, g, l, and m for the surrounding hollow regions as usual.
4. For both cylinder and sphere $\Delta t_1 = m_1$, $\Delta t_2 = m_2$, $\Delta t_3 = m_3$, $\Delta t_{f4} = m_{f4}$.

Heat Transfer across a Fluid-filled Gap. A gap can be treated as an additional layer, say the ith layer. Usually there is no heat generation in the layer and f_i, g_i, and m_i are all zero. The value of l_i is as follows:

	Slab	Cylinder	Sphere
Convection	$1/h' + 1/h''$	$1/(r_i h') + 1/(r_{i+1} h'')$	$1/(r_i^2 h') + 1/(r_{i+1}^2 h'')$
Conduction	a_i/k	$(\ln r_{i+1}/r_i)/k_i$	$(1/r_i - 1/r_{i+1})/k_i$

where h' and h'' are the convection coefficients on the two sides of the gap.

Temperature Distribution within a Region. The temperature distribution within the first layer is given by the general case in Tables 46 to 48. With proper change of subscripts the same formulas, including the criterion for a temperature maximum, apply to any layer.

Meaning of Parameters. The parameter f represents‡ the total heat generation within a layer, l represents the temperature difference across a layer caused by unit rate of constant heat through-flow, gl is the temperature difference across a layer caused by the internal heat generation within the layer, and m is the temperature difference caused by heat generation within the layer and all interior layers.

Simplified Formulas for Fuel Elements. Table 50 gives formulas for simple cases, ignoring heat generation in the clad. For less simple cases, use the general multilayer solution.

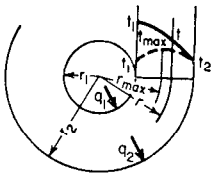
* Δt_{tot} may be positive, zero, or negative. It is positive if the temperature at the left-hand side of a slab is higher than that at the right-hand side, or if the temperature inside a cylinder or sphere is higher than the temperature outside.

† See second footnote in Table 49.

‡ The characteristic geometric divisors A, $2\pi L$, and 4π have been applied to simplify the arithmetic.

Table 47. Conduction in Homogeneous Cylinders

Nomenclature and General Procedure*



Calculate $q_1/2\pi L$ if not already known:
(a) If q_2 is known $q_1/2\pi L = q_2/2\pi L - H_1(r_2^2 - r_1^2)/2$
(b) If $t_1 - t_2$ is known, $q_1/2\pi L$ is given in each particular case by the first equation for that case.
k_1 = thermal conductivity
H_1 = heat generation per unit time per unit volume
For sign conventions, see text.

General Solution for Hollow Cylinder†

See above figure.
Criterion for internal temperature maximum:
cooling from both sides *or*
$r_2^2 - r_1^2 > -(q_1/2\pi L)(2/H_1) > 0$

$$q_1/2\pi L = \frac{(t_1 - t_2)k_1 - H_1[(r_2^2 - r_1^2) - 2r_1^2 \ln r_2/r_1]/4}{\ln r_2/r_1}$$

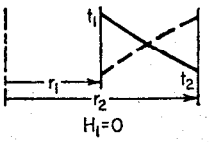
$$t_1 - t_2 = [(q_1/2\pi L) \ln r_2/r_1 + H_1(r_2^2 - r_1^2 - 2r_1^2 \ln r_2/r_1)/4]/k_1$$

$$t_1 - t = [(q_1/2\pi L) \ln r/r_1 + H_1(r^2 - r_1^2 - 2r_1^2 \ln r/r_1)/4]/k_1$$

$$r_{max}^2 = r_1^2 - (q_1/2\pi L)(2/H_1)$$

$$t_1 - t_{max} = H_1 r_1^2(\alpha - 1 - \alpha \ln \alpha)/4k_1 \text{ where } \alpha = (r_{max}/r_1)^2$$

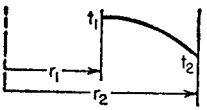
Hollow Cylinder with Heat Through-flow and No Internal Heat Generation



$q_1/2\pi L = (t_1 - t_2)k_1/\ln r_2/r_1$
$t_1 - t_2 = [(q_1/2\pi L) \ln r_2/r_1]/k_1$
$t_1 - t = [(q_1/2\pi L) \ln r/r_1]/k_1$
Highest temperature: t_1 or t_2 at r_1 or r_2, respectively.

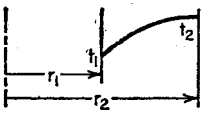
Hollow Cylinder with Internal Heat Generation and No External Heat Input

Case 1. Outward heat flow



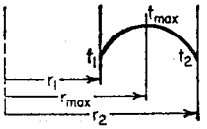
$q_1/2\pi L = 0$
$t_1 - t_2 = H_1[(r_2^2 - r_1^2) - 2r_1^2 \ln r_2/r_1]/4k_1$
$t_1 - t = H_1[(r^2 - r_1^2) - 2r_1^2 \ln r/r_1]/4k_1$
Maximum temperature: t_1 at r_1

Case 2. Inward heat flow



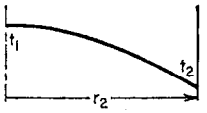
$q_1/2\pi L = -H_1(r_2^2 - r_1^2)/2$
$t_2 - t_1 = H_1[2r_2^2 \ln r_2/r_1 - (r_2^2 - r_1^2)]/4k_1$
$t - t_1 = H_1[2r_2^2 \ln r/r_1 - (r^2 - r_1^2)]/4k_1$
Maximum temperature: t_2 at r_2

Case 3. Outward and inward heat flow, surface temperatures equal



$q_1/2\pi L = -H_1(r_{max}^2 - r_1^2)/2$
$r_{max}^2 = (r_2^2 - r_1^2)/(2 \ln r_2/r_1)$
$t_{max} - t_1 = H_1 r_1^2(1 - \alpha + \alpha \ln \alpha)/4k_1$
where $\alpha = (r_{max}/r_1)^2 = [(r_2/r_1)^2 - 1]/(2 \ln r_2/r_1)$
For $t_1 - t$ use the general formula.

Solid Cylinder with Internal Heat Generation



$t_1 - t_2 = H_1 r_2^2/4k_1$
$t_1 - t = H_1 r^2/4k_1$
$t_{max} = t_1$

---

* Solutions are for uniform heat generation. For exponential heat generation of the form

$$H = (\alpha/r) \exp(-\mu_\alpha r) + (\beta/r) \exp(-\mu_\beta r)$$

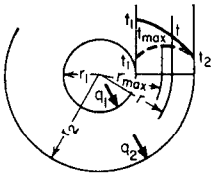
$$t_1 - t = \left\{ (q_1/2\pi L) \ln r/r_1 + \frac{\alpha}{\mu_\alpha} [\exp(-\mu_\alpha r_1) \ln r/r_1 + E_1(\mu_\alpha r) - E_1(\mu_\alpha r_1)] + \frac{\beta}{\mu_\beta} [\exp(-\mu_\beta r_1) \ln r/r_1 + E_1(\mu_\beta r) - E_1(\mu_\beta r_1)] \right\}$$

where E_1 is the exponential integral (Table 15 of Sec. 1-2).

† To take into account unequal surface cooling or heating conditions of a single-layer hollow cylinder, use Table 49 but reject all terms applying to second and third layers.

Table 48. Conduction in Homogeneous Spheres

Nomenclature and General Procedure*



Calculate $q_1/4\pi$ if not already known

(a) If q_2 is known, $q_1/4\pi = q_2/4\pi - H_1(r_2^3 - r_1^3)/3$

(b) If $t_1 - t_2$ is known, $q_1/4\pi$ is given in each particular case by the first equation for that case.

For sign conventions see text.

General Solution for Hollow Sphere†

See above figure.

Criterion for internal temperature maximum:

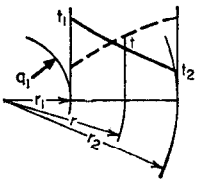
cooling from both sides

or $r_2^3 - r_1^3 > -(q_1/4\pi)(3/H_1) > 0$

$$q_1/4\pi = (t_1 - t_2)k_1/(1/r_1 - 1/r_2) - H_1(r_2 - r_1)(r_2 + 2r_1)r_1/6$$
$$t_1 - t_2 = [(q_1/4\pi)(1/r_1 - 1/r_2) + H_1(r_2 - r_1)^2(r_2 + 2r_1)/6r_2]/k_1$$
$$t_1 - t = [(q_1/4\pi)(1/r_1 - 1/r) + H_1(r - r_1)^2(r + 2r_1)/6r]/k_1$$
$$r_{max}^3 = r_1^3 - (q_1/4\pi)(3/H_1)$$

For t_{max} evaluate r_{max} and substitute for r in $t_1 - t$ formula.

Hollow Sphere with Heat Through-flow and No Internal Heat Generation

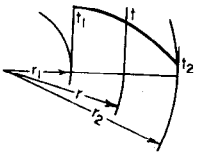


$$q_1/4\pi = (t_1 - t_2)k_1/(1/r_1 - 1/r_2)$$
$$t_1 - t_2 = (q_1/4\pi)(1/r_1 - 1/r_2)/k_1$$
$$t_1 - t = (q_1/4\pi)(1/r_1 - 1/r)/k_1$$

Highest temperature: t_1 or t_2 at r_1 or r_2, respectively.

Hollow Sphere with Internal Heat Generation and No External Heat Input

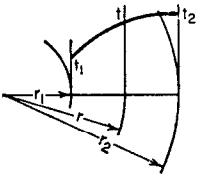
Case 1. Outward heat flow



$$q_1/4\pi = 0$$
$$t_1 - t_2 = H_1(r_2 - r_1)^2(r_2 + 2r_1)/6r_2k_1$$
$$t_1 - t = H_1(r - r_1)^2(r + 2r_1)/6rk_1$$

Maximum temperature: t_1 at r_1

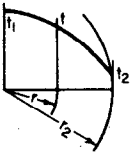
Case 2. Inward heat flow



$$q_1/4\pi = -H_1(r_2^3 - r_1^3)/3$$
$$t_2 - t_1 = H_1(r_2 - r_1)^2(2r_2 + r_1)/6k_1r_1$$
$$t - t_1 = H_1(r - r_1)[2(r_2^3 - r_1^3)/r_1 - (r - r_1)(r + 2r_1)]/6k_1r$$

Maximum temperature: t_2 at r_2

Solid Sphere with Internal Heat Generation

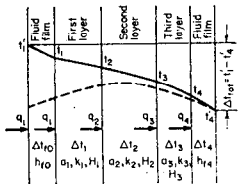
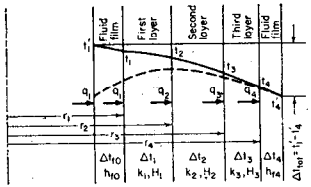
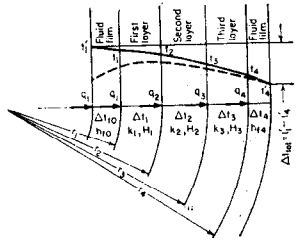


$$t_1 - t_2 = H_1r_2^2/6k_1$$
$$t_1 - t = H_1r^2/6k_1$$
$$t_{max} = t_1 \text{ at center}$$

* Solutions are for uniform heat generation.

† To take into account unequal surface cooling or heating conditions in a single-layer hollow sphere, use Table 49 but reject all terms applying to second and third layers.

Table 49. Multilayer Solids with Heat Generation Uniform in Each Layer and Convection Cooling on Both Sides*

Parameter	Slab or plate	Cylinder	Sphere
			
f_1	H_1a_1	$H_1(r_2^2 - r_1^2)/2$	$H_1(r_2^3 - r_1^3)/3$
f_2	H_2a_2	$H_2(r_3^2 - r_2^2)/2$	$H_2(r_3^3 - r_2^3)/3$
f_3	H_3a_3	$H_3(r_4^2 - r_3^2)/2$	$H_3(r_4^3 - r_3^3)/3$
g_1	$H_1a_1/2$	$H_1[(r_2^2 - r_1^2)/\ln r_2/r_1 - 2r_1^2]/4$	$H_1r_1(r_2 - r_1)(r_2 + 2r_1)/6$
g_2	$H_2a_2/2$	$H_2[(r_3^2 - r_2^2)/\ln r_3/r_2 - 2r_2^2]/4$	$H_2r_2(r_3 - r_2)(r_3 + 2r_2)/6$
g_3	$H_3a_3/2$	$H_3[(r_4^2 - r_3^2)/\ln r_4/r_3 - 2r_3^2]/4$	$H_3r_3(r_4 - r_3)(r_4 + 2r_3)/6$
l_{f0}	$1/h_{f0}$	$1/(h_{f0}r_1)$	$1/(h_{f0}r_1^2)$
l_1	a_1/k_1	$(\ln r_2/r_1)/k_1$	$(1/r_1 - 1/r_2)/k_1$
l_2	a_2/k_2	$(\ln r_3/r_2)/k_2$	$(1/r_2 - 1/r_3)/k_2$
l_3	a_3/k_3	$(\ln r_4/r_3)/k_3$	$(1/r_3 - 1/r_4)/k_3$
l_{f4}	$1/h_{f4}$	$1/(h_{f4}r_4)$	$1/(h_{f4}r_4^2)$
q'_1†	q_1/A	$q_1/2\pi L$	$q_1/4\pi$
m_1	The continuation is identical in all geometries $m_1 = g_1l_1 \quad m_2 = (f_1 + g_2)l_2 \quad m_3 = (f_1 + f_2 + g_3)l_3 \quad m_{f4} = (f_1 + f_2 + f_3)l_{f4}$		
q'_1	$[\Delta_{tot} - (m_1 + m_2 + m_3 + m_{f4})]/(l_{f0} + l_1 + l_2 + l_3 + l_{f4})$		
Δt	$\Delta t_{f0} = q'_1l_{f0} \quad \Delta t_1 = q'_1l_1 + m_1$ $\Delta t_2 = q'_1l_2 + m_2 \quad \Delta t_3 = q'_1l_3 + m_3$ $\Delta t_{f4} = q'_1l_{f4} + m_{f4}$		

$q'_2 = q'_1 + f_1 \qquad q'_3 = q'_1 + (f_1 + f_2) \qquad q'_4 = q'_1 + (f_1 + f_2 + f_3)$

* For explanation and extension to cylindrical and spherical assemblies with a solid core, see text.

† If q_4 is known instead of q_1, $q'_1 = q'_4 - (f_1 + f_2 + f_3)$ where q'_4 is q_4/A, $q_4/2\pi L$, or $q_4/4\pi$ for slab, cylindrical, or spherical geometry, respectively.

Table 50. Simplified Conduction Formulas for Fuel Elements

Case	Temperature differences			
	Fuel	Internal gap	Clad	Surface film
Symmetrically clad plate:				
Cooling symmetric	$H_f a_f^2/8k_f$	No gap	$H_f a_f a_c/2k_c$	$H_f a_f/2h$
Cooling from one side	$H_f a_f^2/2k_f$	No gap	0, $H_f a_f a_c/k_c$	$H_f a_f/h$
Cylinder:				
Solid core, gap, clad	$H_f r_f^2/4k_f$	$H_f r_f a_g/2k_g$	$H_f r_f^2 a_c/2r_c k_c$	$H_f r_f^2/2Rh$

H_f = heat generation per unit time per unit volume of fuel
a_f, a_c, a_g = thickness (radial in cylindrical geometry) of fuel, clad, and gap, respectively
h = film coefficient at surface
r_f, r_c, and R = radius of fuel, mean radius of clad, and outside radius of clad respectively

14.2 Convection, Condensation, and Boiling

Articles 3, 4, and 5 of Sec. 9-3 give comprehensive compilations of formulas for convection, condensation, and boiling, respectively. Table 11 of Sec. 1-1 may be used as an index to these formulas. For turbulent flow of nonmetallic fluids in long circular pipes

$$\mathrm{Nu} = 0.023\ \mathrm{Re}^{0.8}\ \mathrm{Pr}^{1/3} \qquad \text{See Table 3 of Sec. 9-3}$$

14.3 Radiation

In the high-heat-flux regions of reactors, radiation is usually negligible compared with conduction or convection.

14.31 Surface Radiation. For black-body conditions:

$$q/A = 0.171\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right] \qquad \mathrm{Btu/(ft^2)(hr)}$$

$$h_R \approx 6.84\left(\frac{T_{av}}{1000}\right)^3 \qquad \mathrm{Btu/(ft^2)(hr)(°F)}$$

$$q/A \approx h_R(t_1 - t_2) \qquad \mathrm{Btu/(ft^2)(hr)}$$

where T_1 and T_2 = absolute temperatures of the two surfaces, °R
T_{av} = average absolute temperature of the two surfaces, °R
t_1 and t_2 = temperatures of the two surfaces, °F

In cases where surface radiation is important, emissivity and geometric factors must be introduced.*

14.32 Gas Radiation. Carbon dioxide and steam are the only gaseous reactor coolants that have strong absorption (and radiation) wavelength bands. The coefficient of heat transfer is usually at least an order of magnitude lower than for surface radiation, and can be evaluated as a rough approximation from Fig. 1. If the value from Fig. 1 shows gas radiation to be important, a more accurate evaluation should be made.*

Example: For a layer 0.01 ft thick of a mixture of 50 per cent CO_2 with nonabsorbing gas, at a pressure of 100 atmospheres, and average gas-wall temperature of 1500°F:

$$DP = 0.01 \times 50 \times 100 = 50$$

From Fig. 1, h_{GR} = 5 Btu/(ft²)(hr)(°F).

* See, for example, W. H. McAdams, "Heat Transmission," 3d ed., McGraw-Hill Book Company, Inc., New York, 1954.

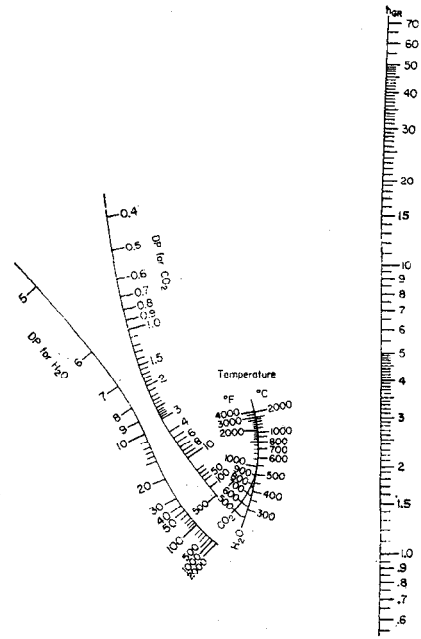


FIG. 1. Nomograph to evaluate heat transfer coefficient by gas radiation. (*From G. Etherington and H. Etherington, "Modern Furnace Technology," 3d. ed., Charles Griffin & Co., Ltd., London, in press.*)

14.4 Over-all Coefficient of Heat Transfer, Conductance

14.41 Plane Geometry with Heat Flow Normal to Surface. The over-all coefficient of heat transfer U for constant heat through-flow q/A is defined by

$$q/A = U(t_1 - t_2)$$

where t_1 and t_2 are the terminal temperatures of the system.

Series Resistances to Heat Flow. For a series of parallel slabs of uniform thickness

$$U = \frac{1}{\Sigma(1/h) + \Sigma(a/k)}$$

where the first term of the denominator is the sum of the reciprocals of all surface coefficients in series and the second term is the sum of the thickness/conductivity ratios of all solid layers in series.

Heat Flow Processes in Parallel. Heat transfer coefficients by independent processes in parallel are additive. For example, the heat transfer coefficient h from an external surface to surroundings at constant temperature is the sum of the convection and radiation coefficients:

$$h = h_C + h_R$$

If h_C is a conductance across a transparent layer involving coefficients h_1 and h_2 at two heat-exchanging surfaces, $h_C = 1/(1/h_1 + 1/h_2)$.

14.42 Cylindrical and Spherical Geometries

$$\text{Cylindrical: } U = 2\pi L/(\Sigma l_f + \Sigma l) \qquad q_1 = U\Delta t$$
$$\text{Spherical: } U = 4\pi/(\Sigma l_f + \Sigma l) \qquad q_1 = U\Delta t$$

where l_f and l are defined in Table 49.

15 THERMAL STRESS AND DISTORTION*

See Sec. 9-4, by R. A. Daane.

Nomenclature and Units†

a = thickness of plate, inside radius of cylinder, in.
b = outside radius of cylinder, in.
d = diameter of cylinder or sphere, in.
E = modulus of elasticity, lb/(in.^{2})
H = heat generation per volume, Btu/(hr)(in.^{3})
k = thermal conductivity, Btu/(hr)(in.)(°F); $k = k'/12$ where k' is in Btu/(hr)(ft)(°F)
t = temperature at a point, °F
α = coefficient of linear thermal expansion, (°F)$^{-1}$
ν = Poisson's ratio
σ = unit normal stress, tension when positive, lb/(in.^{2})

Table 51*a*. Maximum Thermal Stress for Simple Shapes*

Case	Formula for maximum stress	
	Component for uniform heat generation	Component for temperature difference between surfaces
Location of maximum stress	At surface	At surface
Character of maximum stress	Biaxial, equal stresses	Biaxial, equal stresses
Equality of surface stress	Magnitude same at both surfaces except as modified by factors for thick hollow cylinders	
Sign of surface stress	Tensile at both surfaces	Tensile at colder surface, compressive at hotter surface
Solid sphere σ	$\frac{E\alpha H d^2}{60k(1-\nu)}$	
Long solid cylinder σ	$\frac{E\alpha H d^2}{32k(1-\nu)}$	
Flat plate, free to bend σ	$\frac{E\alpha H a^2}{12k(1-\nu)}$	0
Flat plate, not free to bend σ	$\frac{E\alpha}{2(1-\nu)} \cdot \frac{Ha^2}{6k}$	$\frac{E\alpha}{2(1-\nu)} \cdot (t_2 - t_1)$†
Hollow cylinder‡		
Inside stress σ_a	$\frac{E\alpha}{2(1-\nu)} \cdot f_1 \frac{Ha^2}{6k}$	$\frac{E\alpha}{2(1-\nu)} \cdot f_2(t_b - t_a)$
Outside stress σ_b	$\frac{E\alpha}{2(1-\nu)} \cdot f_1 \frac{Ha^2}{6k}$	$\frac{E\alpha}{2(1-\nu)} \cdot f_3(t_b - t_a)$
$\sigma_a - \sigma_b$	0	$\frac{E\alpha}{1-\nu} \cdot (t_b - t_a)$

* Stress components are algebraically additive; a positive value indicates tension at the surface and a negative value indicates compression. For calculation of temperature difference, see Tables 46 to 48.
† t_2 and t_1 are the surface temperatures. The tensile (positive) stress is at the colder surface.
‡ f_1, f_2, and f_3 are given in Table 51*b*. t_a and t_b are the temperatures at the inner and outer surfaces, respectively.

15.1 Order of Magnitude of Thermal Stress Calculated by Elastic Theory

$$\sigma_{\max} = \frac{KE\alpha\Delta t}{1-\nu}$$

where Δt is the greatest temperature difference in the body and K is a function of

* For significance of calculated thermal stress see Art. 1.5 of Sec. 9-4.
† The formulas are valid for any consistent set of units, e.g., CGS°C.

Table 51*b*. Factors *f* for Use with Hollow-cylinder Formulas

ID/OD	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$f_1 = \frac{3}{2}\left[a^2 + b^2 - \frac{b^2 - a^2}{\ln b/a}\right] \Big/ (b^2 - a^2)$	1.07	1.04	1.02	1.01	1.01	1.00	1.00	1.00	1.00	1.00
$f_2 = \frac{2b^2}{b^2 - a^2} - \frac{1}{(\ln b/a)}$	1.59	1.46	1.37	1.29	1.22	1.17	1.12	1.07	1.04	1.00
$f_3 = \frac{2a^2}{b^2 - a^2} - \frac{1}{(\ln b/a)} = f_2 - 2$	−0.41	−0.54	−0.63	−0.71	−0.78	−0.83	−0.88	−0.93	−0.96	−1.00

$\frac{E\alpha}{(1 - \nu)}$ at room temperature (see Table 1 of Sec. 9-4): Uranium (cast) 230, Beryllium (extruded) 300, Graphite 1.8, Austenitic stainless steel 370, Aluminum 210, Zirconium 60, Structural steel 260, Cadmium 210, Concrete 30.

shape and temperature distribution. For simple shapes, K for homogeneous materials is usually between ½ and 1; also $\sigma_{max} \approx \frac{E\alpha\Delta t'}{1 - \nu}$ where $\Delta t'$ is the difference between extreme and average temperatures in the body.

Exception: Axial Temperature Gradient in Long Bar

$$\text{Generally, } \sigma_{max} < \tfrac{1}{2}E\alpha(\Delta t)_d$$

where $(\Delta t)_d$ is the maximum temperature difference in a length equal to the major cross-sectional dimension of the bar.

15.2 Thermal Distortion of Bars and Rods

See Art. 2 of Sec. 9-4.

16 RADIATION DAMAGE TO LIQUIDS AND ORGANIC MATERIALS

See Sec. 10-5, by V. P. Calkins.

16.1 Calculation of Radiation Damage to Specific Materials

See Art. 3 of Sec. 10-5.

Damage to a specific material is estimated from Table 1 of Sec. 10-5 (water, aqueous solutions, and fused salts), Table 2 (organic fluids), Table 3 (elastomers), or Table 4 (plastics). The procedure is as follows:

Step 1. Calculate the rate of energy deposition E_i for the given flux of each kind of particle, using the right-hand part of the appropriate table:

$$E_i = \frac{(\text{given particle flux}) \times 10^n}{\text{Particle dosages each equivalent to } 1 \times 10^n \text{ rads}} \quad \text{rads/sec}$$

For Tables 1 and 2, $n = 9$; for Tables 3 and 4, $n = 7$.

Determine the total rate of energy deposition by adding the values of E_i for each kind of particle.

Step 2. Find the acceptable dose in rads from the damage indicated in the table. Let this be D.

Step 3. Useful life of material $= D/E$ sec, $= D/3{,}600E$ hr $= D/(3.16 \times 10^7 E)$ years. A numerical example is given in Art. 3.2 of Sec. 10-5.

16.2 Estimate of Fraction of Molecules Affected

In absence of data, the fraction of molecules affected may be estimated by formulas for energy absorption given in Art. 2 of Sec. 10-5.

$$\text{Fraction of affected molecules} \approx 10^{-12}GMD$$

where G = molecules affected per 100 ev absorbed [commonly about four for covalent materials (see Art. 2.4 of Sec. 10-5)]
M = molecular weight of compound
D = absorbed dose in rads

For example, if $G = 4$, $M = 100$, and $D = 10^8$ rad, the fraction of molecules affected = 0.04 or 4 per cent.

16.3 Radiation Damage to Electrical, Electronic, and Mechanical Systems

See Table 5 of Sec. 10-5.

17 SOLUTION OF EQUATIONS

See Sec. 3-1, by Alston S. Householder, and Sec. 3-2, by Ward Conrad Sangren.

The theory and solution of equations are discussed in Arts. 4.2 and 4.3 of Sec. 3-1. Practical application of some of the methods of solution is illustrated here.

17.1 Polynomial Equations

17.11 Quadratic Equations

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

17.12 Cubic and Quartic Equations. Cubic equations can be solved algebraically or trigonometrically, and the method can be extended algebraically to solution of quartic (biquadratic) equations that have at least one pair of real roots. However, Horner's method and Graeffe's method, described below, can be conveniently applied to these equations.

17.13 Graeffe's Method for Polynomial Equations. See Art. 4.3 of Sec. 3-1. Graeffe's method is direct, requires no trial solutions, and solves for complex roots as well as real roots. Complications which may develop are discussed at the end of the example.

The Method. To solve the equation $x^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g = 0$, tabulate and proceed as described at the top of page 1–64.

Criteria for Terminating the Process and Formulas for Roots of the Original Equation. CASE 1. Each of the coefficients $b_m, c_m, \ldots$ is equal to the square of the corresponding coefficient for the preceding equation; i.e., the product terms become negligible. This is the case when all roots of the equation are real and unequal.

In absolute magnitude, $x_1 = \sqrt[m]{b_m/1}$, $x_2 = \sqrt[m]{c_m/b_m}$, $x_3 = \sqrt[m]{d_m/c_m}$, etc. The sign for each root must be found by trial, or by applying rules from theory of equations.

CASE 2. The coefficients in a given column, say the c coefficients, become and remain always half the square of the corresponding coefficients of the preceding equations, while the coefficients on either side of this column behave normally, i.e., become the square of the preceding coefficients. This indicates two numerically equal roots, although the signs may be different.

The absolute magnitude of each of the equal roots is $\sqrt[2m]{d_m/b_m}$, i.e., the $2m$th root of the ratio of the normal coefficients.

CASE 3. The coefficients of one column, say the c coefficients, continue to behave irregularly, while the coefficients on either side behave normally. This indicates a pair of complex roots $x = u \pm iv$. A negative coefficient at any stage (except in the original equation) indicates the presence of such a pair of roots.

First stage of solution								Description of process
m								
1	$a = 1$	b	c	d	e	f	g	= coefficients of original equation
	a^2	b^2	c^2	d^2	e^2	f^2	g^2	= squares of coefficients
		$-2ac$	$-2bd$	$-2ce$	$-2df$	$-2eg$		= products as indicated, signs alternating
			$+2ae$	$+2bf$	$+2cg$			
				$-2ag$				
2	1	(algebraic sums of above terms)					g^2	= Coefficients of transformation equation
4								Process is repeated until criteria below are satisfied. For successive values of m, write $m = 1, 2, 4, 8, 16, \ldots$
8								
16								
m	1	b_m	c_m	d_m	e_m	f_m	g_m	

Modulus

$$r^2 = \sqrt[m]{d_m/b_m}$$
$$u = -\tfrac{1}{2}(b + \text{algebraic sum of all roots})$$
$$v = \sqrt{r^2 - u^2}$$
$$x = u \pm iv$$

OTHER CASES. Cases of more than two roots equal in absolute magnitude, or more than one pair of complex roots between two real roots can also be solved by Graeffe's method, as shown on page 1–65.*

Limitations of Graeffe's Method. Columns representing multiple roots may converge slowly, and initial convergence may even give place to divergence as the method seeks to find roots which, in the higher-order equations, have been rendered unequal by accumulation of approximation errors. Roots of approximately equal absolute value also lead to slow convergence. There are simple algebraic processes (see Art. 4.2 of Sec. 3-1) for detecting and eliminating multiple roots (see standard works on theory of equations). Horner's method (Art. 17.14) is useful to find and remove troublesome roots.

Numerical Example from Page 1–65.

FIRST ROOT: $x_1 = \sqrt[m]{b_m/1} = \sqrt[64]{1.6148 \times 10^{33}}$. $\text{Log}\, x_1 = 33.20812 \div 64 = 0.51888$ $|x_1| = 3.3028$. The sign can be found by trial, or often by applying rules from theory of equations; for example, it is shown in Art. 17.14 that there is a root between -3 and -4, and it can easily be shown that there is no root greater than 3. $x_1 = -3.3028$.

PAIR OF EQUAL ROOTS (shown by column c):

$$|x_2| = |x_3| = \sqrt[2m]{d_m/b_m} = \sqrt[2\times 64]{(8.7596 \times 10^{77})/(1.6148 \times 10^{33})} = 2.2361$$

It is found that both positive and negative values satisfy the equation.

* See, for example, Doherty, R. E., and E. G. Keller, "Mathematics of Modern Engineering," vol. 1, pp. 109–128, John Wiley & Sons, Inc., New York, 1936.

Example: Solve $2x^6 + 7x^5 - 8x^4 - 33x^3 - 11x^2 - 10x + 5 = 0$

	a	b	c	d	e	f	g
$m = 1$	1	3.5	−4	−16.5	−5.5	−5	2.5
	1	12.250 8.000	16.00 115.50 −11.00	272.25 −44.00 −35.00 −5.00	30.25 −165.00 −20.00	25.000 27.500	6.2500
$m = 2$	1	20.250	120.50	188.25	−154.75	52.500	6.2500
	1	410.06 −241.00	1.45202×10^4 -0.76241×10^4 -0.03095×10^4	3.5438×10^4 3.7295×10^4 0.2126×10^4 -0.0012×10^4	2.3948×10^4 -1.9766×10^4 0.1506×10^4	2756.3 1934.4	39.063
$m = 4$	1	169.06	0.65866×10^4	7.4848×10^4	0.56883×10^4	4690.7	39.063
	1	2.8581×10^4 -1.3173×10^4	0.43383×10^8 -0.25308×10^8 0.00011×10^8	56.022×10^8 -0.749×10^8 0.016×10^8 —	0.3236×10^8 -7.0218×10^8 0.0051×10^8	0.22003×10^8 -0.00444×10^8	1525.9
$m = 8$	1	1.5408×10^4	0.18086×10^8	55.289×10^8	-6.6931×10^8	0.21559×10^8	1525.9
	1	2.3741×10^8 -0.3617×10^8	0.032710×10^{16} -0.017038×10^{16} —	3056.9×10^{16} 2.4×10^{16} — —	44.798×10^{16} -23.840×10^{16} —	0.046479×10^{16} 0.000204×10^{16}	0.023284×10^8
$m = 16$	1	2.0124×10^8	0.015672×10^{16}	3059.3×10^{16}	20.958×10^{16}	0.046683×10^{16}	0.023384×10^8
		4.0498×10^{16} 0.0313×10^{16}	2.4561×10^{28} -1.2313×10^{28} —	9.3593×10^{38} — — —	439.24×10^{32} -285.63×10^{32} —	21.793×10^{28} —	5.4214×10^{12}
$m = 32$	1	4.0185×10^{16}	1.2248×10^{28}	9.3593×10^{38}	153.61×10^{32}	21.793×10^{28}	5.4214×10^{12}
	1	16.148×10^{32} —	1.50014×10^{56} -0.75221×10^{56} —	87.596×10^{76} — — —	2.3596×10^{68} -4.0793×10^{68} —	4.7493×10^{58} —	29.392×10^{24}
$m = 64$	1	16.148×10^{32}	0.74793×10^{56}	87.596×10^{76}	-1.7197×10^{68}	4.7493×10^{58}	29.392×10^{24}
		Case 1	Case 2		Case 3		

REMAINING REAL ROOT:

$$x_6 = \sqrt[m]{g_m/f_m} = \sqrt[64]{(2.9392 \times 10^{25})/(4.7493 \times 10^{58})} = 0.30277$$

By trial, $x_6 = +0.30277$.

PAIR OF COMPLEX ROOTS:

$$r^2 = \sqrt[m]{f_m/d_m} = \sqrt[64]{(4.7493 \times 10^{58})/(8.7596 \times 10^{77})} = 0.49994$$

$$u = -\tfrac{1}{2}(b + \text{algebraic sum of all real roots})$$
$$= -\tfrac{1}{2}(3.5 - 3.3028 + 2.2361 - 2.2361 + 0.30277) = -0.24999$$

$$v = \sqrt{r^2 - u^2} = \sqrt{0.49994 - 0.06249} = 0.66140$$

$$x_{4,5} = u \pm iv = -0.24999 \pm 0.66140i = 0.24999(-1 \pm 2.6457i)$$

The six roots of the equation are: -3.3028, ± 2.2361, $0.2500(-1 \pm 2.6457i)$, $+0.3028$.

17.14 Horner's Method for Polynomial Equations. See Art. 4.3 of Sec. 3-1. This method is usually used in conjunction with rules for exploring the character of roots and assisting in approximate location of roots. The method is illustrated, without such aids, using the same equation as in the illustration by Graeffe's method.

Step 1. Divide through by the coefficient of the highest power of x, write

$$f(x) = x^6 + 3.5x^5 - 4x^4 - 16.5x^3 - 5.5x^2 - 5x + 2.5$$

and evaluate $f(x)$ for a series of values of x:

x	-4	-3	-2	-1	0	1	2	3
$f(x)$	478.5	-32	10.5	12	2.5	-24	-49.5	748

$f(x)$ can be evaluated by direct substitution, but (except for small integral values of x) it can be evaluated more easily by the remainder theorem. This is illustrated for $x = 3$:

1	3.5	-4	-16.5	-5.5	-5	2.5	$\lfloor 3$
	3	19.5	46.5	90	253.5	745.5	
1	6.5	15.5	30	84.5	248.5	**748**	

Add (vertical arrow) and multiply by 3 (diagonal arrow) as indicated.
$f(x) = 748$

From the change in sign of tabulated values, $f(x)$ becomes zero between $x = -4$ and $x = -3$, between $x = -3$ and $x = -2$, between $x = 0$ and $x = 1$, and between $x = 2$ and $x = 3$.

Step 2. Arbitrarily, the equation will be solved for the root between 2 and 3. A new equation is formed with all its roots 2 less than those of the original equation. The process starts like the method described in Step 1 for evaluating $f(x)$, but is repeated in the manner shown.

1	3.5	-4	-16.5	-5.5	-5	2.5	$\lfloor 2$
	2	11	14	-5	-21	-52	
1	5.5	7	-2.5	-10.5	-26	**-49.5**	
	2	15	44	83	145		
1	7.5	22	41.5	72.5	**119**		
	2	19	82	247			
1	9.5	41	123.5	**319.5**			
	2	23	128				
1	11.5	64	**251.5**				
	2	27					
1	13.5	**91**					
	2						
1	**15.5**						

The "remainders" (boldface) are the coefficients of the transformed equation, $f(x) = x^6 + 15.5x^5 + 91x^4 + 251.5x^3 + 319.5x^2 + 119x - 49.5 = 0$, whose roots are all 2 less than those of the original equation. This equation must therefore have a root between 0 and 1 corresponding to the root of the original equation between 2 and 3. As a first approximation, this root is given by $49.5/119 = 0.416$ (since $x < 1$, terms in higher powers of x tend to become small).

Evaluate $f(x)$ for $x = 0.4$ as described in Step 1.

1	15.5	91	251.5	319.5	119	−49.5	|0.4
	0.4	6.36	38.94	116.18	174.27	117.31	
1	15.9	97.36	290.44	435.68	293.27	**67.81**	

Higher values of x give larger positive values of $f(x)$; so the root must be less than $x = 0.4$. Trial shows that for $x = 0.3, f(x) = 22.52$, and for $x = 0.2, f(x) = -10.76$; i.e., the root lies between $x = 0.2$ and $x = 0.3$. All roots of the equation are next decreased by 0.2.

1	15.5	91	251.5	319.5	119	−49.5	|0.2
	0.2	3.14	18.83	54.07	74.71	38.74	
1	15.7	94.14	270.33	373.57	193.71	**−10.76**	
	0.2	3.18	19.46	57.96	86.31		
1	15.9	97.32	289.79	431.53	**280.02**		
	0.2	3.22	20.11	61.98			
1	16.1	100.54	309.90	**493.51**			
	0.2	3.26	20.76				
1	16.3	103.80	**330.66**				
	0.2	3.30					
1	16.5	**107.10**					
	0.2						
1	**16.7**						

The transformed equation with roots 2.2 less than those of the original equation is

$$x^6 + 16.7x^5 + 107.10x^4 + 330.66x^3 + 493.51x^2 + 280.02x - 10.76 = 0$$

To continue the process, locate the root lying between 0 and 0.1. As a first approximation, $x = 10.76/280.02 = 0.038$, and evaluation confirms that the root lies between 0.03 and 0.04. The equation is therefore transformed by reducing all roots by 0.03, giving

$$x^6 + 16.88x^5 + 109.61x^4 + 343.66x^3 + 523.85x^2 + 310.54x - 1.91 = 0$$

The roots of this equation are now 2.23 less than those of the original equation, and continuation of the process establishes the root of the new equation as being close to 0.006; hence the root of the original equation is

$$x = 2.236$$

Step 3. Reduce the original equation by dividing by $(x - 2.236)$. The synthetic division is identical with Step 1, with the root being used as the multiplier

1	3.5	−4	−16.5	−5.5	−5	2.5	|2.236
	2.236	12.8257	19.7343	7.2319	3.8725	−2.5211	
1	5.736	8.8257	3.2343	1.7319	−1.1275	−0.0211	

The reduced equation is $x^5 + 5.7360x^4 + 8.8257x^3 + 3.2343x^2 + 1.7319x - 1.1275 = 0$. The quantity -0.0211, interpreted in Step 1 as the value of $f(x)$ when $x = 2.236$, is alternatively the remainder after division by $(x - 2.236)$. Ideally the remainder should be zero, and the division serves as a check on the accuracy of the solution.

The reduced equation is solved for another root, and the process can be continued until all real roots have been found. If there is only one pair of complex roots, these can be obtained from the residual quadratic equation.

Solution for the root between 0 and 1 gives $x = 0.304$ and a reduced equation $x^4 + 6.0400x^3 + 10.6619x^2 + 6.4755x + 3.7005 = 0$. Solution of this equation for the root between -2 and -3 gives $x = -2.236$ and a reduced equation

$$x^3 + 3.8040x^2 + 2.1562x + 1.6542 = 0$$

Solution for the root between -3 and -4 gives $x = -3.303$ and a reduced equation $x^2 + 0.5010x + 0.5014 = 0$. Solution of this quadratic equation gives

$$x = 0.2505(-1 \pm 2.6439i)$$

The roots of the equation are thus -3.303, ± 2.236, $0.2505(-1 \pm 2.6439i)$, $+0.304$.

Limitation of Horner's Method. If the equation contains more than one pair of complex roots, the residual equation after eliminating real roots is of fourth or higher degree. The residual equation can, however, be solved by Graeffe's method.

17.2 Solution of Transcendental Equations

The following methods are applicable for finding real roots of any equation in one variable, including transcendental equations. The methods are illustrated by solution of the equation $x - \sin x = 1$.

17.21 Approximate Solution by the Two-graph Method. Let $y = \sin x$, then also $y = x - 1$. Plot the two equations with y as ordinate and x as abscissa. Points of intersection of the two curves give roots of the equation. In this case, the only real root is at $x \approx 1.9$ (Fig. 2). A large-scale plot in the region of $x = 1.9$ will give a closer approximation.

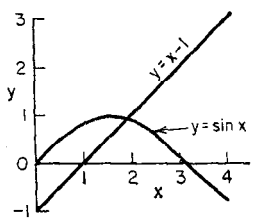


FIG. 2. Two-graph method to solve transcendental equations.

This method is useful as a starting point for the following two numerical methods.

17.22 Method of Successive Linear Interpolation. (See also Method of False Position in Art. 4.3 of Sec. 3-1.) Write $f(x) = x - \sin x - 1$ and find two values of x close together, say a and b, such that $f(a) = a - \sin a - 1$ and $f(b) = b - \sin b - 1$ differ in sign. Then linear interpolation will give as a first approximation

$$x_1 = a - \frac{f(a)}{[f(b) - f(a)]/(b - a)} \qquad \text{or} \qquad x_1 = b - \frac{f(b)}{[f(b) - f(a)]/(b - a)}$$

The process is repeated using values of a and b closer to x_1, but still yielding a difference in sign between $f(a)$ and $f(b)$. Further repetition gives increasingly close approximation.

For the illustrative equation,

$$f(a) = f(1) = 1 - 0.8415 - 1 = -0.8415$$
$$f(b) = f(2) = 2 - 0.9093 - 1 = +0.0907$$
$$\theta_1 = 1 - \frac{-0.8415}{(0.0907 + 0.8415)/(2 - 1)} = 1.9027$$

or

$$\theta_1 = 2 - \frac{0.0907}{(0.0907 + 0.8415)/(2 - 1)} = 1.9027$$

For the next approximation try $a = 1.90$, $f(a) = -0.0463$, and (after inspecting the values developed in the first approximation) $b = 1.95$, $f(b) = +0.0210$.

$$\theta_2 = 1.9 - \frac{-0.0463}{(0.0210 + 0.0463)/(1.95 - 1.90)} = 1.9344$$

For the next approximation, try $a = 1.934$, $f(a) = -0.0008$, and (maintaining a difference of sign) $b = 1.935$, $f(b) = 0.0006$.

$$\theta_3 = 1.934 - \frac{-0.0008}{0.0014/0.001} = 1.934 + 0.0006 = 1.9346$$

The next approximation gives $\theta_4 = 1.9345$.

17.23 Newton's Method. Newton's method is a refinement of the method of successive linear interpolation. It is illustrated by solution of the equation used in the linear-interpolation method above.

Write $f(\theta) = \theta - \sin\theta - 1$ and the first and second derivatives. $f'(\theta) = 1 - \cos\theta$, $f''(\theta) = \sin\theta$.

As before, locate the root approximately: $f(1) = -0.8415$, $f(2) = +0.0907$. Find $f''(1) = 0.8415$, $f''(2) = +0.9031$.

For a first approximation, θ_1, use the value of θ in which $f(\theta)$ and $f''(\theta)$ have the same sign, i.e., $\theta = 2$.

$$\theta_2 = \theta_1 - \frac{f(\theta_1)}{f'(\theta_1)} = 2 - \frac{0.0907}{1.4161} = 1.9359$$

Successive approximations are made without reference to $f''(\theta)$.

$$\theta_3 = \theta_2 - \frac{f(\theta_2)}{f'(\theta_2)} = 1.936 - \frac{0.0020}{1.3571} = 1.9346$$
$$\theta_4 = 1.9346 - \frac{0.0001}{1.3558} = 1.9345$$

Newton's method is convenient only if the derivative is a simple expression; also, convergence is contingent on specific conditions being satisfied. The less elegant linear-interpolation method is not so restricted.

17.3 Solution of Systems of Linear Algebraic Equations

17.31 Solution by Elimination. Several variants of this method are discussed in Art. 4.1 of Sec. 3-1, including the Doolittle method. This method is illustrated by the following numerical example, incorporating an arithmetic check at each step:

$$4.15x - 3.17y + 2.41z = 7.61$$
$$3.21x + 2.12y - 3.31z = 2.41$$
$$2.91x + 1.65y + 2.33z = 8.41$$

Operation	x	y	z	Right side	Check sum* By operation	Check sum* By addition
1. Coefficients of equations	4.15	−3.17	2.41	7.61	(11.00)	11.00
	3.21	2.12	−3.31	2.41	(4.43)	4.43
	2.91	1.65	2.33	8.41	(15.30)	15.30
2. Divide each line of 1 by the coefficient of x	1	−0.764	0.581	1.834	2.651	2.651
	1	0.660	−1.031	0.751	1.380	1.380
	1	0.567	0.801	2.890	5.258	5.258
3. Referring to 2, subtract second line from first, and third from first		−1.424	1.612	1.083	1.271	1.271
		−1.331	−0.220	−1.056	−2.607	−2.607
4. Divide each line of 3 by the coefficient of y		1	−1.132	−0.761	−0.893	−0.893
		1	0.165	0.793	1.959	1.958
5. Subtract second line of 4 from first line of 4			−1.297	−1.554	−2.852	−2.851
6. Divide 5 by coefficient of z			1	1.198	2.199	2.198
			z = **1.198**			
From operation 4	$y = 1.132z - 0.761 = (1.132 \times 1.198) - 0.761 =$ **0.595**					
From operation 2	$x = 0.764y - 0.581z + 1.834$ $= (0.764 \times 0.595) - (0.581 \times 1.198) + 1.834$ $=$ **1.593**					

* The numbers in the first column (except numbers in parentheses) are obtained by applying the indicated operation to the preceding check sums. The numbers in the second column (and the numbers in parentheses in the first column) are obtained by direct addition of the coefficients in the same horizontal row. The numbers must check at each line.

Check by substituting in one of the original equations. For example, a check using the first equation gives $(4.15 \times 1.593) - (3.17 \times 0.595) + (2.41 \times 1.198) = 7.612$. It is possible to solve the problem with one less row of figures, but the procedure is somewhat less straightforward.

17.32 Solution by Determinants. See Art. 1.4 of Sec. 3-1. The solution to the simultaneous equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

is written directly in determinant form as follows:

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \qquad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}.$$

The denominator is the same in each case, and is the determinant formed by the coefficients of the unknowns. The numerators are identical with the denominators except for one column, in which the quantities on the right-hand side of the equation replace the coefficients of the particular unknown expressed.

To evaluate the denominator

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

(In this expansion the elements of the first column become the coefficients of the minors, with the signs alternating. Each minor is obtained from the determinant by suppressing the column and row containing the particular a coefficient.)

$$= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

(To evaluate a second-order determinant, multiply the elements diagonally in pairs, downward to the right positive, upward to the right negative.)

For n simultaneous equations, the nth-order determinant is expanded in terms of its n minors (determinants of order $n - 1$), and so on. The properties of determinants permit manipulation to simplify the work.

Numerical example:

$$\begin{aligned} 4.15x - 3.17y + 2.41z &= 7.61 \\ 3.21x + 2.12y - 3.31z &= 2.41 \\ 2.91x + 1.65y + 2.33z &= 8.41 \end{aligned}$$

$$x = \frac{\begin{vmatrix} 7.61 & -3.17 & 2.41 \\ 2.41 & 2.12 & -3.31 \\ 8.41 & 1.65 & 2.33 \end{vmatrix}}{\begin{vmatrix} 4.15 & -3.17 & 2.41 \\ 3.21 & 2.12 & -3.31 \\ 2.91 & 1.65 & 2.33 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} 4.15 & 7.61 & 2.41 \\ 3.21 & 2.41 & -3.31 \\ 2.91 & 8.41 & 2.33 \end{vmatrix}}{\begin{vmatrix} 4.15 & -3.17 & 2.41 \\ 3.21 & 2.12 & -3.31 \\ 2.91 & 1.65 & 2.33 \end{vmatrix}}$$

$$z = \frac{\begin{vmatrix} 4.15 & -3.17 & 7.61 \\ 3.12 & 2.12 & 2.41 \\ 2.91 & 1.65 & 8.41 \end{vmatrix}}{\begin{vmatrix} 4.15 & -3.17 & 2.41 \\ 3.21 & 2.12 & -3.31 \\ 2.91 & 1.65 & 2.33 \end{vmatrix}}$$

$$\begin{aligned} \text{Denominator} &= 4.15 \begin{vmatrix} 2.12 & -3.31 \\ 1.65 & 2.33 \end{vmatrix} - 3.21 \begin{vmatrix} -3.17 & 2.41 \\ 1.65 & 2.33 \end{vmatrix} + 2.91 \begin{vmatrix} -3.17 & 2.41 \\ 2.12 & -3.31 \end{vmatrix} \\ &= 4.15(4.940 + 5.462) - 3.21(-7.386 - 3.977) + 2.91(10.493 \\ &\qquad - 5.109) = 95.311 \end{aligned}$$

By similar evaluation of the numerators

$$x = \frac{151.823}{95.311} = 1.593 \qquad y = \frac{56.774}{95.311} = 0.596 \qquad z = \frac{114.194}{95.311} = 1.198$$

Substituting in original equations

$$\begin{aligned} (4.15 \times 1.593) - (3.17 \times 0.596) + (2.41 \times 1.198) &= 7.609 \\ (3.21 \times 1.593) + (2.12 \times 0.596) - (3.31 \times 1.198) &= 2.413 \\ (2.91 \times 1.593) + (1.65 \times 0.596) + (2.33 \times 1.198) &= 8.410 \end{aligned}$$

18 ATOMIC ENERGY COMMISSION LITERATURE

This article is intended as an aid in locating and identifying the AEC literature referenced at the end of most sections of the handbook. References to commercial publications and publications of scientific and technical societies need no clarification.

18.1 Availability of AEC Literature

18.11 AEC Depository Libraries. Unclassified and declassified AEC reports are available in approximately 70 depository libraries throughout the United States.

Approximately the same number of depository libraries have been established in foreign countries. The locations of these libraries are given in most of the general and informational AEC publications, e.g., Refs. 1, 2, and 5 of Art. 18.2.*

A few of the depository libraries are designated "industrial information depository libraries." These have supplementary technical reports and abstracts and collections of unclassified AEC drawings.

18.12 The Office of Technical Services, U.S. Department of Commerce, Washington 25, D.C. Many unclassified and declassified reports can be purchased from this agency (abbreviated OTS in subsequent references) in printed, photostatic, or microcopy form. List of reports free on request (Ref. 3 of Art. 18.21).

18.13 Superintendent of Documents, U.S. Government Printing Office, Washington 25, D.C. Some of the more substantial AEC documents can be purchased from this agency only (abbreviated GPO in subsequent references). List of reports, "Atomic Energy and Civil Defense Price List," free on request.

18.14 AEC Technical Information Service Extension (TISE), P.O. Box 1001, Oak Ridge, Tenn. Some documents are obtainable from this agency (abbreviated TISE in subsequent references).

18.15 Microcopy. Microcopy is available from OTS, from Microcard Foundation, P.O. Box 2145, Madison 5, Wis., and from Readex Microprint Corporation, 115 University Place, New York 3, N.Y., the two latter being commercial enterprises.

18.16 Atomic Energy Publications by Other Public Agencies. United States Bureau of Standards handbooks relating to radiological protection may be obtained from GPO at prices ranging from $0.15 to $0.25.

Nuclear Science Series Reports are available from the National Academy of Sciences—National Research Council, Washington, D.C.

18.2 Guides to AEC Literature

The following AEC publications indicate content, source, price, and other pertinent data relating to official publications.

18.21 Consolidated Listings

1. *Nuclear Science Abstracts* (*NSA*), issued twice monthly, GPO, $7.50 per year, also by purchase of separate issues. Abstracts of pertinent scientific and technical literature, official and unofficial. Also periodic cumulated and supplementary indexes, the latter serving to supplement TID-4000 (see next item).

2. TID-4000, "Cumulated Numerical List of Available Unclassified U.S. Atomic Energy Commission Reports," revised from time to time, OTS, $1.25. Gives report numbers (not titles), readily available sources (official and other), reference to *Nuclear Science Abstracts*, and prices if for sale.

3. "List of AEC Research Reports for Sale," revised twice a year, OTS, free. Gives report numbers, titles, etc., and prices.

4. TID-4022, "Consolidated Availability Listing of Confidential Reports Announced to the Civilian Application Program," revised quarterly, TISE, free to L- and Q-cleared access permit holders. TID-4022 is unclassified. Formerly the listings appeared in the abstract journal, *Confidential Reports for Civilian Applications* (RCCA). Gives report number (not title), category, abstract reference, prices of printed reports and microcopy.

5. TID-4023, "Consolidated Availability Listing of Secret Reports Announced to the Civilian Application Program," revised quarterly, TISE, free to Q-cleared access permit holders. TID-4023 is unclassified. Formerly the listings appeared in TID-3063, "Bibliography of Secret U.S. Atomic Energy Reports Available through the Civilian Application Program." Gives report number (not title), category, abstract reference, prices of printed reports and microcopy.

6. TID-4100, "Unclassified Engineering Materials List" (UEML), with supplements, issued from time to time, Technical Information Service Extension (TISE), P.O. Box 1001, Oak Ridge, Tenn., free.

* An up-to-date list should be consulted, since the number of libraries is continually increasing.

This document is "A Catalog of Drawings, Photographs, and Specifications Released by the United States Atomic Energy Commission." The listed items are for sale and the price and source are stated. Additional information from TISE.

7. TID-4550, "What's Available in the Unclassified Atomic Energy Literature," revised from time to time, OTS, free. Gives general information on AEC literature. Lists some AEC-sponsored series, manuals, and handbooks. Less detailed than TID-4575.

8. TID-4563, "Special Sources of Information on Isotopes," September, 1957, Isotopes Extension Division of Civilian Application, U.S. AEC, Oak Ridge, Tenn., free. A guide to sources of information on radioisotopes.

9. TID-4575, "Guide to Atomic Energy for the Civilian Application Program," revised from time to time, OTS, free. Describes in considerable detail the various categories of AEC publications, including availability and means of locating material.

10. "Selected Readings on Atomic Energy," revised from time to time, GPO, $0.25. Describes important official and commercial books, periodicals, and bibliographies.

11. Classified Reports. Abstracts and consolidated availability listings of confidential and secret reports are available to qualifying organizations and personnel through the Civilian Application Program. These include TID-3063 (bibliography, see Item 5); TID-3080, *Nuclear Technology* (bibliography); TID-3081, *Feed Materials* (bibliography); *Report Announcement Bulletin* (confidential); *Confidential Reports for Civilian Applications*, CRCA (abstract journal, formerly *Civilian Applications of Atomic Energy*).

Changes of classification from secret or confidential to unclassified are listed in TID-4035, "AEC Reports Declassified," and supplements. TID-4035 gives report number, title, author, date, and classification status.

12. Card Catalog. Catalog cards are available (including abstracts) from TISE for reports abstracted in the various AEC journals.

18.22 Additions to the Literature

1. *Nuclear Science Abstracts* (see Ref. 1 of Art. 18.21). Each semimonthly issue gives abstracts of additions to the technical literature, and announces new nuclear data.

2. Monthly listings and abstracts of new literature (unclassified and classified) are available on a subscription basis.

3. Press releases. Important announcements are made by AEC in the form of press releases, which are also given wide distribution in mimeographed form. Many of these announcements are of economic or technical importance.

18.3 References Available in Alternative Forms

Several publications are referred to frequently in the handbook, but not always in identical terms, since some are available in alternative forms. The following are the more important cases of dual publication.

18.31 Nuclear Data. The standard reference for nuclear data other than data on neutron cross sections has been:

K. Way, et al, "Nuclear Data," *Nat. Bur. Standards (U.S.) Circ.* 499 (1950) and supplements.

This important reference is being revised and expanded, under the sponsorship of the U.S. AEC, by the Nuclear Data Group of the National Academy of Sciences—National Research Council. It is now issued as;

a. "Nuclear Level Schemes," published in sections, each section being released when completed, available from GPO.

b. Nuclear Data Cards, available on an annual subscription basis from Publications Office, National Academy of Sciences—National Research Council, Washington, D.C.

c. New Nuclear Data, cumulated from *Nuclear Data Cards* quarterly, semiannually, and annually, and published in *Nuclear Science Abstracts;* the same medium was used formerly to supplement *Nat. Bur. Standards (U.S.) Circ.* 499.

18.32 Reference Books. Several important books have been sponsored by the AEC and published both commercially in cloth covers and also officially in paper

covers. The Government Printing Office editions are often referred to by code numbers instead of titles. The most important books in this category are given below with the name of the commercial publisher and the AEC document number.

1. D. J. Hughes and J. A. Harvey, "Neutron Cross Sections," McGraw-Hill Book Company, Inc., New York, 1955. BNL-325. The preceding edition appeared as AECU-2040.

2. "The Reactor Handbook," published in three volumes. McGraw-Hill Book Company, Inc., New York, 1953. AECD-3645, AECD-3646, and AECD-3647.

18.4 The United States Atomic Energy Commission

1. Semiannual Reports of the U.S. Atomic Energy Commission to the Congress of the United States, Washington, D.C., republished for sale under a variety of titles,* GPO, various prices, $0.35 to $1.25.

Appendixes give much general information and describe the organization, policies, operations, etc., of AEC.

REFERENCES

1. Hughes, D. J., and J. A. Harvey: "Neutron Cross Sections," McGraw-Hill Book Company, Inc., New York, 1955. Also Supplement 1 by D. J. Hughes and R. B. Schwartz (1957).
2. Westcott, C. H.: "Effective Cross Section Values for Well-moderated Thermal Reactor Spectra," CRRP-680, TNCC(Can)-7, (also AECL-407), January 25, 1957. Revised as AECL No. 670(CRRP-787), August 1, 1958.
3. ANL 5800: "Reactor Physics Constants," August, 1958.

* In recent years, for the designated 6-month period the title "Major Activities in the Atomic Energy Programs" has been used.